Nuclear Chemistry Cumulative Examination
January 21, 2015

The examination focuses on radioactive decay kinetics. It consists of 15 questions worth a total of 100 points. Support all answers with brief justifications where appropriate. A periodic table is available on the wall at the front of the room.

Some helpful relations:

$$A = \lambda N \qquad T_{1/2} = \ln 2/\lambda \quad 1\,\mathrm{Ci} = 3.7\times 10^{10}\ \mathrm{dps} \qquad N_A = 6.02\times 10^{23}\mathrm{mol}^{-1}$$

For members of a radioactive decay chain, the general solution for the number of atoms of any member of the chain was provided by H. Bateman. The number of parent and daughter nuclei at any time t is given as:

Parent: $N_1^t = N_1^0 e^{-\lambda_1 t}$ Daughter: $N_2^t = \dfrac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + N_2^0 e^{-\lambda_2 t}$

Q1 (5 points): For the parent, determine N_1^t / N_1^0 when $t = T_{1/2}$.

Q2 (5 points): For the parent, how many decay half-lives must pass for N_1^t to equal 1% of N_1^0?

Q3 (5 points): What is the activity of a 0.114-ng sample of ^{24}Na, a β^- emitter with a half-life of 14.95 h.

Q4 (5 points): If all of the ^{24}Na in Q3 above was bound in a NaCl crystal lattice at the expected 1:1 molar ratio, what would be the specific activity of the sample? The molecular weight of Cl is 35.5 g/mol.

For Q5 through Q8, consider the case of non-equilibrium decay, where $(\lambda_1 >> \lambda_2)$.

Q5 (15 points) On a single Cartesian coordinate plane, graph N_1^t, N_2^t, and total number of nuclei as a function of time. Be sure to clearly label the axes and each of the three curves.

Q6 (5 points) Determine the time when N_1^t has its maximum value.

Q7 (5 points) Determine the time when N_2^t has its maximum value.

Q8 (5 points) Determine the time when $N_1^t = N_2^t$.

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For Q9 through Q12, consider the case of secular equilibrium decay, where $(\lambda_1 << \lambda_2)$.

Q9 (15 points) On a single Cartesian coordinate plane, graph A_1^t, A_2^t, and total activity as a function of time. Be sure to clearly label the axes and each of the three curves.

Q10 (5 points) Determine the time when N_1^t has its maximum value.

Q11 (5 points) Determine the time when N_2^t has its maximum value.

Q12 (10 points): For the case of secular equilibrium, show that $A_1^t = A_2^t$ for large values of *t*.

For Q13 and 14, consider the production of a radioisotope by irradiation, where the activity of a radioisotope at the end of an irradiation can be expressed as $A_1^t = \lambda_1 N_1^t = R\left(1 - e^{-\lambda_1 t}\right)$, where *R* is the production rate and *t* is the irradiation time.

Q13 (5 points): Sketch A_1^t as a function of irradiation time.

Q14 (5 points): Justify why an irradiation time of order 3 times the half-life is sufficient to maximize the activity of the product nuclide.

Q15 (5 points): At the completion of the irradiation, how many decay half-lives must pass for the activity to decrease to 1% of its maximum value?