Nuclear Chemistry Cumulative Examination
Wednesday, 22 October 2o14

This examination consists of twenty questions on general information and properties of nuclei and nuclear reactions. The exam has a total of 100 points.

1. (25 points, 5 each) Provide **concise** and accurate answers to the following five general questions about the properties of nuclei and decay.

   (a) Approximately how many nuclei are stable against radioactive decay (within 10%)?
   just under 300 stable isotopes depending on how you count since a few of which are very long-lived

   (b) Write a **completely** balanced equation for the β^- decay of ${}^{14}_{6}$C. Be sure to include all emitted particles and any net charges on the products. This nucleus is critical for radiological dating of organic materials.
   $\mathbf{{}^{14}_{6}C \rightarrow {}^{14}_{7}N^{+} + \beta^{-} + \overline{\nu} + Q_{\beta-}}$

   (c) Write a **completely** balanced equation for the β^+ decay of ${}^{18}_{9}$F. Be sure to include all emitted particles and any net charges on the products. This nucleus is used extensively in PET imagining in Nuclear Medicine.
   $\mathbf{{}^{18}_{9}F \rightarrow {}^{18}_{8}O^{-} + e^{+} + \nu + Q_{\beta+}}$
   or$\mathbf{{}^{18}_{9}F \rightarrow {}^{18}_{8}O + e^{-} + e^{+} + \nu + Q_{\beta+}}$

   (d) ${}^{238}_{92}$U is radioactive with a relatively long half-life and after a series of β^- and α decays turns into (only) one of the four stable lead isotopes [${}^{204}_{82}$Pb, ${}^{206}_{82}$Pb, ${}^{207}_{82}$Pb, ${}^{208}_{82}$Pb]. Which isotope does it form and why is it easy to predict which one without knowing the details of the decay chain?
   Note that alpha decay will decrease the mass by four units, beta decay leaves the mass unchanged. Thus, 238/4 = 59 + 2 the lead isotope has to have A = n*4 + 2. The answer, A=206 = 51*4 + 2

   (e) People have estimated that approximately 90 PBq (2.4 MCi) each of ${}^{134}_{55}$Cs ($T_{1/2}$=2.0 yr) and ${}^{137}_{55}$Cs ($T_{1/2}$=30 yr) were released in the Fukushima disaster in March, 2011 and that the average level of ground contamination in California from this event was on the order of 100 Bq/m^2. Make an estimate of the ground contamination due to the lighter isotope, ^{134}Cs, in California in March, 2015. [see: http://cerea.enpc.fr/en/fukushima.html]
   The time for decay of ${}^{134}_{55}$Cs is very nearly 2 half-lives, thus the activity should be 1/4 of the original. Activity(${}^{134}_{55}$Cs) = 100 Bq/m^2 / 4 = 25 Bq/m^2, the information about ${}^{137}_{55}$Cs is superfluous.

2. (25 points, 5 points each) Give a concise and accurate answer to each of the following short questions about the structure of very light nuclei.

   (a) There are only two conceivable values for the nuclear spin of a deuteron (^{2}H). What are the two values of the nuclear spin? One spin state is the ground state and the other is the lowest-lying excited state. Given that deuterium is NMR active, what is the ground state nuclear spin of deuterium?
   The unlike nucleons will each be in an s-orbital but they can be spin-paired or not because they are different types of nucleons.
   I(deuteron) = S(proton=1/2 $\hbar$) +/- S(neutron=1/2 $\hbar$)
   thus I = 0 or 1 $\hbar$ only.
   ground state has to be I = 1 because I=0 cannot be NMR active.

   (b) The expected ground state nuclear spins of the dineutron (^{2}n) and ^{2}He are equal. What is the value of their ground state spins and why.
   The dineutron and ^{2}He has a pair of neutrons or a pair of protons, respectively. Thus, they will both have I=0 since the two nucleons will have to be spin-paired to both be in the lowest s-orbital.

   (c) The excited state of the deuteron with the dineutron (^{2}n) and ^{2}He make up a isospin triplet. Two partners in this group are mirror nuclei, which two are they? What is the defining property of mirror nuclei?
   The dineutron and ^{2}He are the mirror nuclei in this set. Mirror pairs of nuclei have the reversed numbers of neutrons and protons. $^{3}_{1}H_2$ and $^{3}_{2}He_1$ is another mirror pair.

   (d) The dineutron (^{2}n) and ^{2}He are both unbound. What, if any, implication does this fact have for the first excited state of the deuteron? Explain.
   The members of an isospin multiplet (in this case a triplet) have identical nuclear wave functions with a small correction for the difference in Coulomb energy due to the differing number of protons. Since the two "outer" members of the multiplet are unbound, one should expect that the central state which lies in-between the other two in energy should also be unbound. Thus, the excited state of the deuteron should be unbound (and it is!).

   (e) The plan for "hot fusion" reactors of the future is to fuse two isotopes of hydrogen (deuterium ^{2}H and tritium ^{3}H) to generate energy. Write a **completely** balanced reaction for this fusion process showing the final products. Does this process lead to any radioactive products?
   $\mathbf{^{2}_{1}H_1 +^{3}_{1} H_2 \rightarrow^{4}_{2} He_2 +^{1} n_1 + Q}$
   The neutron is radioactive ... Note that ^{5}He is unbound, there are no bound nuclei with A=5 (or A=8).

3. (25 points, 5 points each) Give a concise and accurate answer to each of the following short questions about the average properties of heavy nuclei.

   (a) Nuclei with A>~30 have *approximately* the same average binding energy per nucleon, regardless of it being a proton or neutron. What is the numerical value of this average binding energy per nucleon (within 10%)?
   the value is ~8.5 MeV per bound nucleon

   (b) Explain why the heaviest nuclei are unstable with respect to the emission of an alpha particle but the light nuclei are not.
   The nucleus with the highest binding energy has A=56. Thus, it is possible for heavy nuclei to increase their binding energy by decreasing the number of nuclei by splitting (fission). In addition the alpha particle (^{4}He, a doubly-closed shell nucleus) has an unusually high binding energy and heavy nuclei can make a net gain by emitting an alpha particle and decreasing their mass.

   (c) The liquid drop model is often used to describe the average properties of heavy nuclei. The mass density (mass/volume) and the number density (number/volume) of a drop of liquid are constants. Use the known properties of large nuclei to write an expression for the number density of a large nucleus. Simplify the expression, do you get a constant?
   number density = number of nucleons per unit volume, ρ_n= A / V
   Recall that the radius of a nucleus is R = $r_0 A^{1/3}$,
   so for a sphere, V = $(4/3)\pi R^3 = (4/3)\pi r_0^3 A$
   ρ_n = A / $(4/3)\pi r_0^3 A$ = 3 / $4\pi r_0^3$ .. a constant.

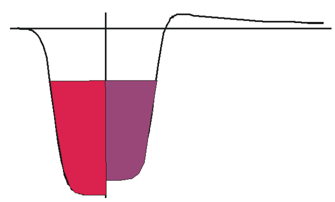
   (d) The semi-empirical mass equation relies on the liquid drop model of the nucleus and only has five contributions (only five adjustable parameters). Indicate the theoretical basis for (or, at least, name) each of the five mathematical terms in the semi-empirical mass equation.
   Empirically the nuclear mass is proportional to (1) number of nucleons (A), with subtractive corrections for (2) number of nucleons on surface ($A^{2/3}$), (3) a Coulomb correction for number of protons (approximately Z^2), (4) a term related to the nucleon asymmetry (N-Z) , and (5) a subtraction for the number of unpaired nucleons (0, 1 or 2).

   (e) The fission process leads to two product nuclei, of course. What is the most-likely fission fragment partner to the pesky ^{134}Cs isotope mentioned above? Explain your reasoning; will the partner have the same or different chemical behavior to cesium when released?
   The nuclear fission occurring in reactors occurs via:
   $^{235}_{92}U +^1 n \rightarrow^{236} U \rightarrow twofissionproducts$

Since Cesium has Z=55, its partner will be 92-55=37 which is Rubidium. Hah, same chemical group !!
The mass number will be approximately 236-134=102 except for 2 or 3 prompt neutrons that are also emitted.

4. (25 points, 5 points each) Give a concise and accurate answer to each of the following short questions about the nuclear structure based on the shell model. (For reference, an energy level diagram is given below.)

   (a) The shell model relies on filling neutrons and protons into separate potential energy wells that use so-called Woods-Saxon potential energy curve for the average nuclear reaction. Make a reasonably accurate sketch of the total potential energy well for neutrons and separately for protons in a heavy nucleus. Indicate a typical value for the depth of the potential well in MeV.
   neutrons on the left, protons on the right, depth approximately 30MeV, color only for emphasis of filling in stable nuclei.



   (b) What is the configuration of neutrons and protons predicted by the shell model for ^{7}Li? And what is the predicted ground state spin and parity of this nuclide?
   3 protons, $\mathbf{1s^2_{1/2}\ 1p^1_{3/2}}$
   4 neutrons, $\mathbf{1s^2_{1/2}\ 1p^2_{3/2}}$
   Only one unpaired nucleon, the proton, in a j=3/2 p-orbit.
   Thus I = 3/2, and parity = -1 (from p, $\ell = 1$)

   (c) The lowest-lying excited state in ^{7}Li is the only bound excited state in this nucleus. What do you expect for the spin and parity of this sole excited state according to the shell model?
   smallest amount of energy in this situation is to promote the proton in the 1p3/2 orbit up to the 1p1/2 orbit and leave the neutrons alone. Thus, the excited state should be I = 1/2 and parity = -1

   (d) The shell model predicts that mirror nuclei will have the same nuclear structure after correction for the difference in Coulomb energy between the pair of nuclei. What is the mirror nucleus for ^{7}Li? Will the ground state of the mirror be shifted up or down in energy compared to the ground state of ^{7}Li?
   The mirror to $^{7}_{3}Li_4$ has to be A=7 with Z=4, or $^{7}_{4}Be_3$.
   ^{7}Be has more protons in the nucleus than ^{7}Li and so its ground state

should be shifted up in energy.
Oooh, I think this is related to question # 2c.

(e) Fact 1: Most of the chemical elements with an odd atomic number have only one stable isotope. Fact 2: Hydrogen, Lithium, Boron and Nitrogen are the only exceptions. Explain these facts using the shell model.
Fact 1: The nucleons are assumed to fill either the proton well or the neutron well and not cross-populate. Thus, all the protons will pair up and we can have either zero or one unpaired proton in any nucleus. Similarly, only zero or one unpaired neutron. Chemical elements with an odd atomic number must have one unpaired proton. Having both an unpaired proton and an unpaired neutron in the same nucleus is very unfavorable with respect to binding energy. In addition, as nuclei become larger the Coulomb energy shifts the relative positions of the proton and neutron orbitals and beta decay becomes favorable.
Fact 2: There are two important factors: Note that the only such nuclei are the lightest members, ${}_{1}^{2}$H, ${}_{3}^{6}$Li, ${}_{5}^{10}$B and ${}_{7}^{14}$N, they each have an unpaired proton and neutron. These nuclei also have equal number of neutrons and protons (which is favored by symmetry). Their Coulomb energy is small due to the small number of protons and the beta decay energy for conversion of a proton into a neutron (or vice versa) is also very small.