Nuclear Chemistry Cumulative Examination
January 22, 2014

This exam focuses on the recent confirmation of the production of element 115 [D. Rudolph *et al*., Phys. Rev. Lett. 111, 112502 (2013)]. The exam will be graded out of 100 points, with the distribution indicated at the start of each question.

Potentially useful constants and equations are provided on the last page of the exam.

I. **(35 points)** *Production of element 115*

The fusion-evaporation reaction $^{48}_{20}Ca + ^{243}_{95}Am$ was used to produce isotopes of element 115.

a. Identify the compound nucleus formed in this reaction.
b. Calculate the Coulomb barrier, in units of MeV, for the above reaction.
c. Sketch the behavior of the total reaction cross section as a function of energy for the ^{48}Ca + ^{243}Am reaction. Please label all axes.
d. Estimate the maximum geometrical cross section, in barns, for the reaction.
e. Calculate the center-of-mass energy of the ^{48}Ca + ^{243}Am reaction, assuming a ^{48}Ca ion beam energy of 260 MeV.
f. Determine the Q value for the reaction, given the mass defects for ^{48}Ca, ^{243}Am, and the compound nucleus are –44.2234 MeV, +57.1774 MeV, and +182.77 MeV, respectively.
g. Estimate the excitation energy of the compound nucleus, again assuming a ^{48}Ca ion beam energy of 260 MeV in the laboratory frame of reference.

II. **(25 points)** *^{243}Am target properties*

The $^{243}Am_2O_3$ used for the target in the study is radioactive, with ^{243}Am decaying 100% by alpha emission with a half-life of 7,370 years. The target thickness was 0.83 mg/cm^2.

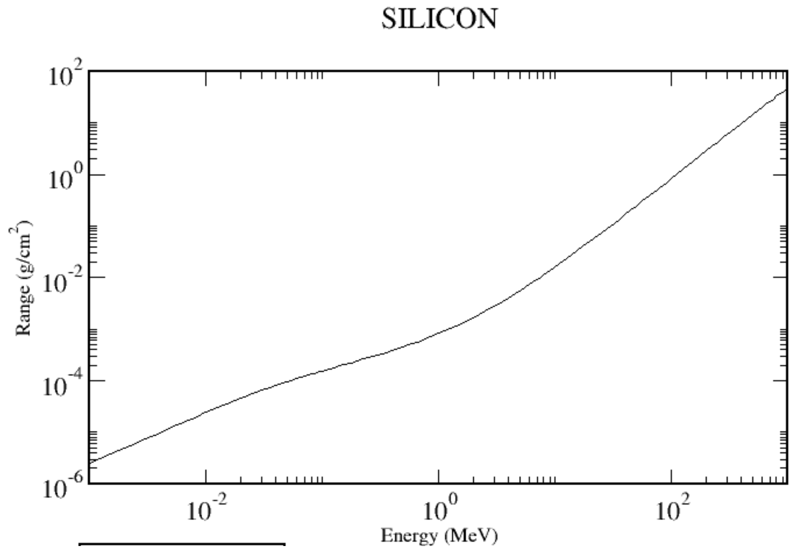
a. Give the complete, balanced equation for the alpha decay of ^{243}Am.
b. Estimate the weight % of Am in the target.
c. Calculate the alpha activity of the target. The density of Am is 13.69 g/cm^3 and the active area of the target is 24 cm^2. Assume that ^{243}Am is the only source of alpha particles from the target.
d. Estimate the percent target loss due to radioactive decay over the course of the experiment, which was completed in 15 days.

III. **(20 points)** *^{288}115 decay properties*

Element 115 was observed to decay by alpha-particle emission with a half-life of 0.17 s.

a. The alpha particle energy detected following $^{288}115$ decay was ~10 MeV. Estimate the energy of the alpha particle using the Geiger-Nuttall rule:
$\log t_{1/2} = -46.83 + 1.454 \times Z/Q^{1/2}$, where Q is in MeV and $t_{1/2}$ in seconds.

b. The range of alpha particles in silicon is shown in the accompanying figure. Estimate the minimum thickness (in mm) of silicon required to detect the full energy of ~10 MeV alpha particles emitted from $^{288}115$. The density of Si is 2.33 g/cm^3.



c. Sketch the expected alpha particle energy spectrum in 1.0 mm of Si from the decay of $^{288}115$. Be sure to label all axes.

d. The $^{288}115$ decay chain terminates at $^{268}Db_{105}$, which undergoes spontaneous fission. Assuming $^{268}Db_{105}$ undergoes asymmetric fission, sketch the expected charge distribution of the fission fragments. Again, be sure to label all axes.

IV. **(20 points)** *K x-ray detection*

An array of germanium photon counters surrounded the silicon alpha detectors and were used to identify x rays emitted along the $^{288}115$ decay chain.

a. Why the interest in obtaining an x-ray "fingerprint" of the elements undergoing decay?

b. Why were germanium detectors selected for the K x-ray measurements?

c. Estimate the $K_{\alpha 2}$ energy for element 115, assuming a hydrogen-like atom and given the $K_{\alpha 2}$ energy for Pb is ~72.8 keV.

d. How are the K x rays generated during the decay process?

Constants and Equations

$c = 2.998\times10^{8}\, m\cdot s^{-1}$ $h = 6.626\times10^{-34}\, J\cdot s$

$\hbar = h/2\pi = 1.055\times10^{-34}\, J\cdot s$ $k_B = 1.380\times10^{-23}\, J\cdot K^{-1}$

$e = 1.602\times10^{-19}\, C$ $N_A = 6.022\times10^{23}\, mol^{-1}$

$1\, amu = 1.660\times10^{-27}\, kg = 931.5016\, MeV$ $m_e = 9.11\times10^{-31}\, kg = 0.511\, MeV$

$m_H = 1.673\times10^{-27}\, kg = 938.7906\, MeV$ $m_n = 1.6749\times10^{-27}\, kg = 939.5731\, MeV$

$1\, ev = 1.602\times10^{-19}\, J$ $1\, fm = 10^{-15}\, m$ $1\, angstom = 10^{-10}\, m$

$1\, b = 10^{-24}\, cm^2$ $1\, Ci = 3.7\times10^{10}\, dps = 3.7\times10^{10}\, Bq$

1 yr = 365 d = 8760 h = 5.256×10^{5} min = 3.1536×10^{7} sec

$R = r_0 A^{1/3},\ r_0 = 1.2\,\mathrm{fm}$ $A = N\lambda$

$\lambda = \dfrac{0.693}{t_{1/2}}$ $\beta = v/c$

$c = \lambda\nu$

$B_{tot}(A,Z) = [Z\cdot M({}^1H) + N\cdot M(n) - M(A,Z)]c^2$ $B_{ave}(A,Z) = B_{tot}(A,Z)/A$

$\Delta(A,Z) = M(A,Z) - A$

$N_1 = N_0 e^{-\lambda_1 t}$ $N_2(t) = \dfrac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + N_2^0 e^{-\lambda_2 t}$

$V_C = 1.44\,\mathrm{fm}\cdot\mathrm{MeV}\left(\dfrac{Z_1 Z_2}{R}\right)$ $V_\ell = \dfrac{\ell(\ell+1)\hbar^2}{2\mu R^2}$

$\mu = (m_1 m_2)/(m_1 + m_2)$

$T_{cm} = (m_p + m_T)v_0^2/2$ $T_0 = T_P - T_{cm} = T_P\left(m_T/(m_T + m_P)\right)$

$U = \dfrac{m_T}{m_T + m_P}T_P + S_P;\ U = T_{cm} + Q$ $T_p = T_{cm}\left((m_p + m_T)/m_p\right)$

$\sigma_{total} = \pi R^2\left(1 - \dfrac{B}{T_P}\right)$ $TKE = \dfrac{Z_1 Z_2 e^2}{1.8\left(A_1^{1/3} + A_2^{1/3}\right)}$

$$-\frac{dE}{dx}_{Bethe-Bloch} = 0.3070\frac{MeVcm^2}{g}\rho\frac{Z}{A}\frac{q^2}{\beta^2}\left[\ln\frac{W_{\max}}{I} - \beta^2\right]$$

$$R_2(T_2) = \frac{M_2}{M_1}\frac{q_1^2}{q_2^2}R_1\left(T_2\frac{M_1}{M_2}\right)$$