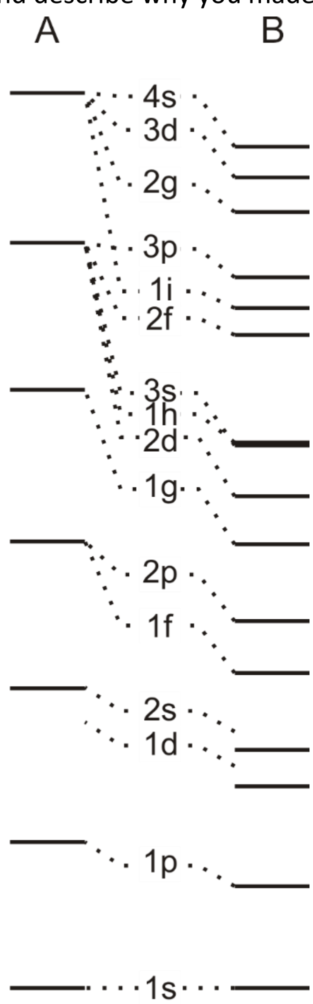
Nuclear Chemistry
Cumulative Exam
Wed 16th, 2013

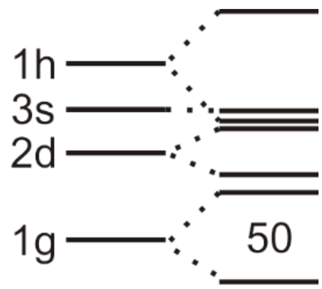
This exam is focused on the nuclear shell model. The exam will be graded out of 90 points with the breakdown indicated for each question.

1. (20 points) Consider the three nuclear potentials (1) infinite square well, (2) harmonic oscillator, and (3) Woods-Saxon.
   a. Sketch the three nuclear potentials in your test book as a function of radius.
   b. Single particle level diagrams are shown in the figure below using two different nuclear potentials A and B. In your test book label A and B as either the infinite square well or harmonic oscillator potential and describe why you made that choice?



A – harmonic oscillator, B – infinite square well. All *l* orbitals within a major harmonic shell are degenerate.

2. (25 points) Reproduce the following figure in your exam book. The right hand side of the figure shows the proton single-particle orbitals in the vicinity of the tin isotopes and includes the effect of the spin-orbit interaction.



a. Label all seven levels on the right-hand side of the figure with *l* (use the notation s,p,d,f …) and j.

    From lowest to highest $g_{9/2}$, $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $h_{11/2}$, $s_{1/2}$, $h_{9/2}$

b. Why does the 3s state not split into two separate levels?

    $l = 0$

c. The expectation of the spin-orbit operator ($l*s$) can be obtained using the following operator:

    $(l*s) = 0.5*(j^2 - l^2 - s^2)$

    Based on the level diagram shown above what is the sign of the spin-orbit interaction?

    $(l*s)|\Psi\rangle = 0.5\,[j(j+1) - l(l+1) - s(s+1)]\ |\Psi\rangle$

    For a given *l* there are two possible orientations of s. $j1 = l + s$ and $j2 = l - s$.

    (1) $(l*s)|\Psi_{j1}\rangle = 0.5\,[(l+s)(l+s+1) - l(l+1) - s(s+1)]\ |\Psi_{j1}\rangle$
    (2) $(l*s)|\Psi_{j2}\rangle = 0.5\,[(l-s)(l-s+1) - l(l+1) - s(s+1)]\ |\Psi_{j2}\rangle$

    The value of j_1 is larger than j_2. A negative sign on the spin-orbit force would lower the higher j configuration.

d. List the expected spins and parities of the ground state and first three excited states of $^{133}_{51}Sb_{82}$.

    $7/2^+$, $5/2^+$, $3/2^+$, $11/2^-$

e. What ground state spins and parities are possible based on the coupling of an odd-proton with an odd-neutron in $^{134}_{51}Sb_{133}$ assuming the odd neutron is placed in the $f_{7/2}$ orbital?

    Possible spin values range from $|j_2-j_1|$ through j_2+j_1. The neutron is in the $f_{7/2}$ orbital and the proton is in the $g_{7/2}$. Spin can range from 0 to 7 with negative parity.

f. What do you expect for the ground state spin and parity of $^{134}_{51}Sb_{83}$?

    0^-, the odd nucleons are both particles and one is in an orbital with $j = l + ½$ and the other is in an orbital with $j = l - ½$.

3. (40 points) The following questions (a-h) refer to the Ca isotopic chain and focus strictly on neutrons in the $f_{7/2}$ single-particle orbit.
    a. What is the degeneracy of the $f_{7/2}$ orbital?

        8 nucleons

    b. What are the allowed J values for two neutrons in the $f_{7/2}$ orbit? (Hint: Start by determining all allowed combinations of m_1 and m_2, the projections of j_1 and j_2 on the z-axis, and then group the results into states with total J).

        Construct a table of J and projection of J along z-axis, M.

M_1	M_2								M_J
	-7/2	-5/2	-3/2	-1/2	1/2	3/2	5/2	7/2	
-7/2		x							-6
			x						-5
				x					-4
					x				-3
						x			-2
							x		-1
								x	0
-5/2			X						-4
				X					-3
					X				-2
						X			-1
							X		0
								X	1
-3/2				X					-2
					X				-1
						X			0
							X		1
								X	2
-1/2					X				0
						X			1
							X		2
								x	3
1/2						X			2
							X		3
								X	4
3/2							X		4
								X	5
5/2								X	6

Based on the projections shown in the last column there must be one state with J = 6 (shaded green) followed by one state with J = 4 (shaded blue), one state with J = 2 (shaded red) and lastly, one state with J = 0. So the allowed spins are J = 0,2,4,6.

c. What is the ground state spin and parity of ${}^{42}_{20}Ca_{22}$ and why?

The ground state spin and parity of ${}^{42}Ca$ (and all nuclei with an even number of protons and neutrons) is 0^+ due to the pairing force.

d. Using the binding energies provided in the table at the end of this test to estimate the neutron effective single-particle energy of the $f_{7/2}$ orbital.

The difference in binding energies between ${}^{41}Ca$ and ${}^{40}Ca$ can be considered as the effective single-particle energy of a neutron in the $f_{7/2}$ orbital.

40Ca Binding Energy = 40*8551.301 keV/A = 342052 keV

41Ca Binding Energy = 41*8546.703 keV/A = 350414 keV

Difference = 8362 keV

e. Based on the answer to part (c) what would you predict for the binding energy of ${}^{42}Ca$?

In the absence of any interactions between the two neutrons the binding of 42Ca should be

42Ca Binding Energy = 342052 + 2*8362 = 358776

42Ca Binding Energy/A = 8542.2

f. Use the discrepancy between the answer for part (e) and the binding energy of ${}^{42}Ca$ in the table to estimate the two body matrix element between two neutrons in the $f_{7/2}$ single particle orbit coupled to J = 0.

Discrepancy amounts to 358776 – (42*8616.559) = -3119.478 keV

g. The first excited 2^+, 4^+, and 6^+ states in ${}^{42}Ca$ are at 1524, 2752, and 3189 keV. Estimate the energies of the two body matrix elements for two neutrons in the $f_{7/2}$ single particle orbit coupled to J=2.

358776 – (42*8616.559) + 1524 = -1595 keV

h. What J values are expected for ${}^{43}Ca$ considering only three neutrons in the $f_{7/2}$ orbital?

M1	M2	M3								MJ
		-7/2	-5/2	-3/2	-1/2	1/2	3/2	5/2	7/2	
-7/2	-5/2			X						-15/2
					X					-13/2
						x				-11/2
							X			-9/2
								X		-7/2
									X	-5/2
-7/2	-3/2				X					-11/2
						X				-9/2
							X			-7/2
								X		-5/2
									X	-3/2
-7/2	-1/2					X				-7/2
							X			-5/2
								X		-3/2

									X	-1/2
-7/2	1/2						X			-3/2
								X		-1/2
									X	1/2
-7/2	3/2							X		1/2
									X	3/2
-7/2	5/2								x	5/2
-5/2	-3/2				X					-9/2
						X				-7/2
							X			-5/2
								X		-3/2
									X	-1/2
-5/2	-1/2					X				-5/2
							X			-3/2
								X		-1/2
									X	1/2
-5/2	1/2						X			-1/2
								X		1/2
									x	3/2
-5/2	3/2							X		3/2
									X	5/2
-5/2	5/2								X	7/2
-3/2	-1/2					X				-3/2
							X			-1/2
								x		1/2
									x	3/2
-3/2	1/2						X			1/2
								X		3/2
									X	5/2
-3/2	3/2							X		5/2
									X	7/2
-3/2	5/2								X	9/2
-1/2	1/2						X			3/2
								X		5/2
									X	7/2
-1/2	3/2							X		7/2
									X	9/2
-1/2	5/2								x	11/2
1/2	3/2							X		9/2
									X	11/2
1/2	5/2								X	13/2
3/2	5/2								x	15/2

The allowed J values are 15/2, 11/2, 9/2, 7/2, 5/2, 3/2.

	proton	neutron	40Ca	41Ca	42Ca	43Ca	44Ca
Mass Excess	8.071	7.289	-34.847	-35.138	-38.544	-38.405	-41.466
Mass (amu)			39.962591	40.962278	41.958618	42.958766	43.955481
Conversions	1 amu = 931.5 MeV/c2						
Atomic mass (amu)	1.007825	1.008665	39.96259	40.962278	41.958618	42.958766	43.955481
Binding energy/nucleon (keV)	0	0	8551.301	8546.703	8616.559	8600.659	8658.17
			-304484.8				