Nuclear Chemistry
Cumulative Examination

Wednesday, 27 February 2o13

Write your answers to the following questions *in the order listed.* Make sure that the answers are well-organized and self-explanatory. The total number of points on this exam is 50.

1. (2 point each) Give concise and accurate answers to the following questions:
   (a) What is the key feature of two nuclides that are said to have the same "isospin projection?"
   isospin projection = (N-Z)/2 thus they have the same neutron excess
   (b) What is the key feature of two nuclides that are said to be"isomers?"
   nuclear isomers are excited states of a given nuclide
   (c) What is the relationship between the partial half-life and (full) half-life of a radionuclide?
   the partial half-life refers to a specific decay branch of a nuclide.
   partial half-life = branching ratio * (full) half-life
   (d) Write a COMPLETELY balanced equation for the β^+ decay of the nuclide ^{22}Na (Z=11, a nuisance activity with a half life of 2.60 years that is often produced when accelerated beams strike aluminum).
   $^{22}\mathbf{Na} \rightarrow {}^{22}\mathbf{Ne} + e^- + e^+ + \nu + Q$
   (e) The *lepton number* is conserved in nuclear decay. What is a lepton in this context?
   The positron and the neutrino are both leptons and are emitted from the nucleus. FYI: the "extra" electron was present before the decay in the neutral sodium atom.

2. (5 points each) The ^{132}Sn nuclide is something of a special nuclide because it is strongly produced in the fission of uranium, it is a so-called doubly-magic nucleus, and it decays with a half-life of 39.7s. The ground state intrinsic spin and parity of ^{132}Sn is 0^+ and it decays with a Q-value of 3119 keV to the radioactive nucleus ^{132}Sb that has a ground-state intrinsic spin/parity of 4^+. This beta decay takes place via an allowed Gammow-Teller transition between the initial and final states.
   (a) What are the intrinsic spins and their relative alignment for the particles that are emitted from the nucleus in an allowed Gammow-Teller β^- decay?
   In this decay an electron and an antineutrino are emitted, both are s=1/2, and the spins are aligned to a total S=1.
   (b) Would you expect this beta decay to go directly from the ground state of the parent to the ground state of the daughter nucleus? Explain why or why not.
   Parent or initial (ground) state I=0 and daughter ground state I=4. Since ΔS=1 for G-T decay, then angular momentum coupling would require at least ΔL=2$\hbar$ to reach the daughter's ground state. For an "allowed G-T decay" the ΔL=0 so the final state has to be an excited state with I=1.
   (c) Suppose that some of the β^- decay goes to an excited state. What is the most likely decay mode of this state and how will the lifetime of this state compare to the β^- decay

lifetime of the parent?
The most likely decay mode of an excited state in ^{132}Sb is gamma decay moving to a lower energy state in the same nucleus. This decay would be much faster than the beta decay, perhaps on the order of pico or even femto seconds.

(d) The beta decay of the daughter nucleus ^{132}Sb has a significantly larger Q-value of 5508 keV even though it is "closer to stability." This is an example of a general phenomena in the beta decay of nuclei with even mass numbers. Explain the basis for why the decay of this daughter has a larger Q-value than it's parent decay.
All even mass number beta-decay chains alternate between even-even nuclei (e.g., ^{132}Sn) and odd-odd nuclei (e.g., ^{132}Sb). Thus, the Sn beta decay will convert a fully paired even-even nucleus into an doubly unpaired odd-odd nucleus. This has a large energy COST. The second Sb beta decay will convert an odd-odd nucleus into a fully paired even-even nucleus with a significant energy release. The energy difference of two times the pairing energy will be found in decays along all even-A beta decay chains.

3. (10 points each) The A=132 mass chain accounts for 4.3% of the yield of fission fragments from the thermal-neutron fission of ^{235}U and 4.95% from thermal-neutron fission of ^{233}U.

(a) (i) What is meant by the term "thermal neutron?" (ii) Give a detailed explanation of why these nuclei can undergo fission with a thermal neutron whereas the much more abundant uranium isotope, ^{238}U, does not.
A thermal neutron has a kinetic energy of 3/2 k_BT .
The fissile (even-odd) uranium (Z=92) nuclei capture a neutron to make even-even nuclei. This releases slightly more energy than the capture of a neutron by ^{238}U which forms an even-odd nucleus. The difference of one unit of pairing energy is sufficient to put the even-even nuclei over the barrier to fission while the even-odd nucleus remains below it.

(b) Use conservation of (1) momentum, (2) mass and (3) energy to make an estimate of the kinetic energy of an A=132 fission fragment from the fission of ^{236}U. For this estimate you can ignore neutron emission, assume that the fissioning nucleus is at rest, and the energy released by the fission process is 200 MeV.

(1) Conservation of momentum $\rightarrow m_1v_1 + m_2v_2 = 0$ **or** $v_1 = (m_2v_2/m_1)$
(2) Conservation of mass: $\rightarrow m_1 + m_2 = 236amu$
(3) Conservation of energy: $\rightarrow KE_1 + KE_2 = m_1v_1^2/2 + m_2v_2^2/2 = 200MeV$
substitute for v_1 in (3):
$m_1(m_2v_2/m_1)^2/2 + m_2v_2^2/2 = 200MeV$
$(m_2/m_1)m_2v_2^2/2 + m_2v_2^2/2 = 200MeV$
$[(m_2/m_1) + 1]m_2v_2^2/2 = 200MeV$
$[(m_2 + m_1)/m_1]m_2v_2^2/2 = 200MeV$
$m_2v_2^2/2 = 200MeV * m_1/(m_1 + m_2)$
Substitute in masses:
$KE_2 = m_2v_2^2/2$ = 200 MeV * (236 - 132) / 236 = 88 MeV