Nuclear Chemistry
Cumulative Exam
Wed. 19th 2012

This exam focuses on nuclear masses and binding energies. The exam will be graded out of 90 points with the point distribution indicated for each question.

1. A simple semiempirical mass equation for predicting the mass of a nucleus can be written as

$$M(Z,A)c^2 = [Z*M(^1H) + N*M(n)]c^2 - B_{tot}$$

The total binding energy of a nucleus with Z protons and a mass number A can be written as:

$$B_{tot}(Z,A) = a_1A – a_2A^{2/3} – a_3Z^2/A^{1/3} – a_4(A-2Z)^2/A \pm a_5/A^{1/2}$$

Where the constants a_1, a_2, a_3, a_4, a_5 have the values 15.56 MeV, 17.23 MeV, 0.7 MeV, 23.28 MeV, and 11 MeV, respectively.

a. (15 points) Describe the physical significance of each of the five terms in the binding energy equation.
   i. a_1A – volume term
   ii. $a_2A^{2/3}$ – surface term
   iii. $a_3Z^2/A^{1/3}$ – coulomb term
   iv. $a_4(A-2Z)^2/A$ – asymmetry term
   v. $a_5/A^{1/2}$ – pairing term

b. (5 points) Based on the equation above explain the gradual shift toward larger N/Z ratios in stable nuclei as A increases.
   i. As Z increases, the increasing magnitude of the Coulomb energy is compensated by deviating from N = Z to N > Z in heavier nuclei. See the chapter on nuclear properties in the textbook Modern Nuclear Chemistry by Loveland, Morrissey and Seaborg.

c. (5 points) What percentage of the total binding energy is accounted for by the surface term for $^{40}_{20}Ca$ and $^{238}_{92}U$?
   i. B.E. ($^{40}_{20}Ca$) = $15.56 * 40 – 17.23 * 40^{(2/3)} – 0.7 * 20^2/40^{1/3} – 23.28 * (40 – 2*20)^2/40 – 11 /40^{1/2}$ = 337 MeV
   Surface contribution = $17.23 * 40^{(2/3)}$ = 201 MeV
   Surface / Total = 60%
   ii. B.E. ($^{238}_{92}Ca$) = $15.56 * 238 – 17.23 * 238^{(2/3)} – 0.7 * 92^2/238^{1/3} – 23.28 * (238 – 2*92)^2/238 – 11 /238^{1/2}$ = 1799 MeV
   Surface contribution = $17.23 * 238^{(2/3)}$ = 661 MeV
   Surface / Total = 37%

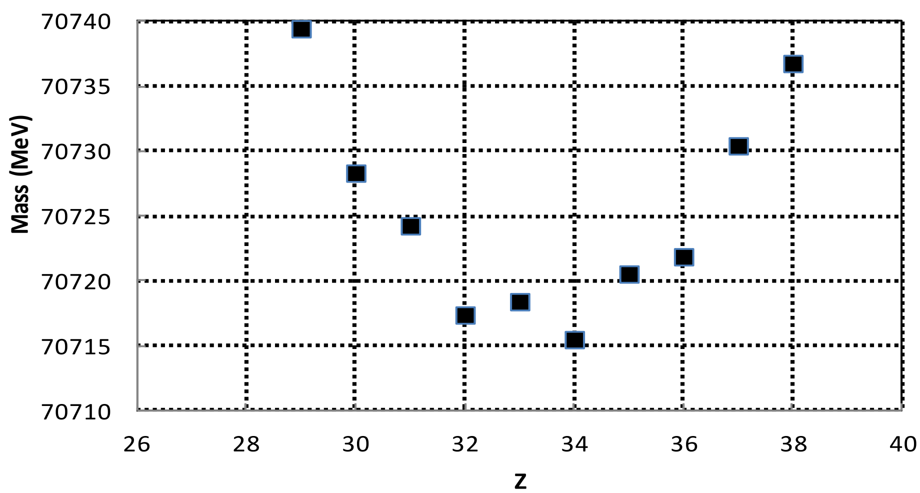
d. (5 points) Estimate the energy released when $^{238}_{92}U$ fissions into two equal mass nuclei neglecting pairing.
   i. Energy released = B.E.(A,Z) – 2*B.E.(A/2,Z/2)
   Volume and asymmetry do not contribute to the difference so
   Energy released = $– 17.23 * A^{2/3} – a_3Z^2/A^{1/3} – 2*[– 17.23 * (A/2)^{2/3} – a_3(Z/2)^2/(A/2)^{1/3}]$

For Z = 92 and A = 238, energy released is 180 MeV

e. (5 points) What is the mass of $^{40}_{20}Ca$ in amu using the following additional data; M(1H) = 1.00728 amu, M(n) = 1.00867 amu, the binding energy per nucleon for $^{40}_{20}Ca$ is 8.5513 MeV, 1 amu = 1.6606 x 10^{-27} kg, c = 2.99 x 10^{8} m/s, 1 J = 6.24151 x 10^{12} MeV.

   i. B.E. = 40 * 8.5513 MeV = 342.052 MeV
   Convert binding energy into mass units using E = mc^2
   m = E/c^2 = 342.052 MeV * (1J/6.24151x10^{12} MeV) / (2.99 x 10^8 m/s)^2 = 6.129994 * 10^{-28} kg
   m = 6.129994 * 10^{-28} kg * (1 amu/1.6606 x 10-27 kg) = 0.3691 amu
   M($^{40}_{20}Ca$) = 20(1.00728 amu) + 20(1.00876 amu) – 0.3691 amu
   M($^{40}_{20}Ca$) = 39.96 amu

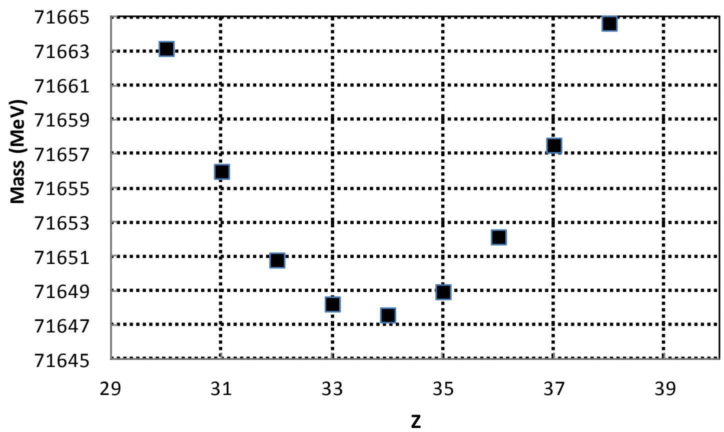
2. Shown below are the mass parabolas for the mass 76 nuclei ($_{29}Cu$, $_{30}Zn$, $_{31}Ga$, $_{32}Ge$, $_{33}As$, $_{34}Se$, $_{35}Br$, $_{36}Kr$, $_{37}Rb$, $_{38}Sr$).



a. (4 points) Which isotope(s) would you expect to be stable?
   i. $^{76}_{32}Ge$, $^{76}_{34}Se$
b. (4 points) Which isotope(s) may decay by both β^+/EC and β^- decay?
   i. $^{76}_{33}As$
c. (4 points) For which isotope(s) may double beta decay be experimentally observed?
   i. $^{76}_{32}Ge$
d. (4 points) Explain the origin of the alternating large and small differences between neighboring masses.
   i. The alternating large and small differences between neighboring masses results from the pairing term which is alternatively positive and negative for odd-odd and even-even nuclei, respectively.
e. (4 points) Derive a rough estimate for the pairing energy based on the mass parabola.
   i. The two parabolas (one for odd-odd nuclei and one for even-even nuclei) are separated by twice the pairing energy. From the plot, the difference between

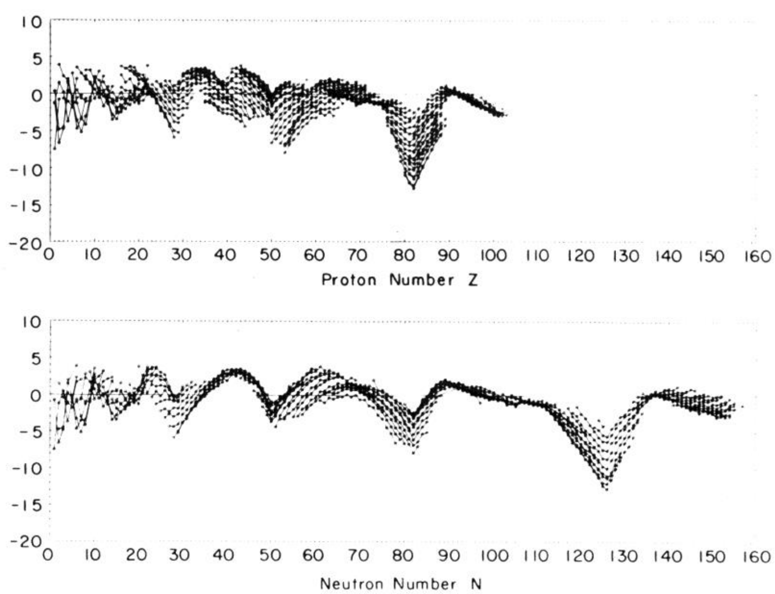
the two plots is ~3 MeV resulting in an estimate for the pairing energy of 1.5 MeV.

3. (5 points) Sketch the expected mass versus Z plot for an odd-A chain of isotopes. How and why does this plot differ from the one presented in question 2 for an even-A system?
    a. Plot for mass 77 nuclei as an example. Plot differs from an even-A system due to lack of oscillating positive and negative pairing term. Only one parabola is present and for a given odd-A mass there is only one stable isotope.



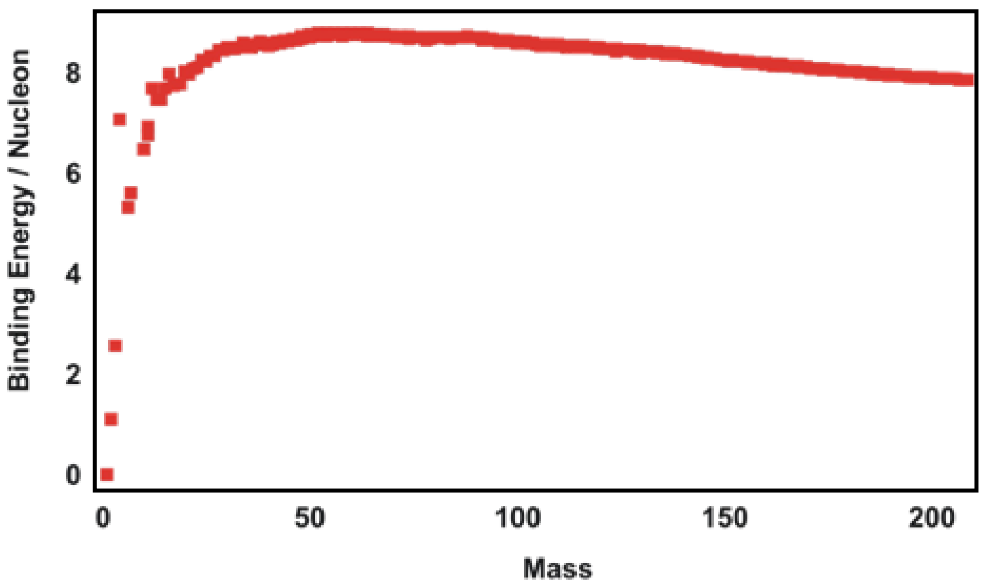
4. (10 points) The figure below shows the deviation between measured and predicted masses (using the equations in question 1) as a function of mass number. Provide a simple explanation to account for the observed deviations.

Mass difference (MeV)



Observed discrepancies can be attributed to the filling of nuclear shells.

5. Shown below is the binding energy per nucleon as a function of mass.
   a. (5 points) What does the relatively flat binding energy per nucleon for A>10 tell you about the nuclear force?
      i. The nuclear force is short ranged and saturates.
   b. (5 points) Determine the mass at which the binding energy per nucleon peaks using the equation for total binding energy presented in question 1 under the assumption that Z = A/2 and no pairing.
      i. $B.E.(Z,A) = a_1A - a_2A^{2/3} - a_3Z^2/A^{1/3}$
         $B.E.(Z,A)/A = a_1 - a_2A^{-1/3} - a_3A^{2/3}/4$
         $d[B.E.(Z,A)/A]/dA = 0 = (a_2/3)*A^{-4/3} - (a_3/6)A^{-1/3}$
         $(a_3/6)A^{-1/3} = (a_2/3)*A^{-4/3}$
         $A = -(a_2 * 2/a_3)$
         $A = (17.23 * 2 / 0.7) = 50$



6. He-3 is used in thermal neutron capture detectors and undergoes the following nuclear reaction:

$$^3He + n \rightarrow {}^3H + {}^1H$$

   a. (5 points) Determine the energy of the emitted triton (^3H) from the following data.
      i. Calculate Q value of reaction
         Q = 14.931 + 8.071 – 14.950 – 7.289 = 0.763 MeV
         Energy is shared between ^3H and ^1H
         Energy of triton = ¼ * 0.763 = 0.19 MeV
   b. (5 points) Is the reaction spontaneous?
      i. The reaction is spontaneous (positive Q value)

The following data may be useful

	Mass Excess, Δ(MeV)		Mass Excess, Δ(MeV)
^{3}He	14.931	^{3}H	14.950
n	8.071	^{1}H	7.289