NUCL Exam 11/28/12 Key

I.

Force	Exchange Boson	Range(m)	Rel. Strength	Time (s)
Strong	gluon (+1)	10^{-15} m (+1)	1	10^{-23} (+1)
Weak	$W^{\pm}, Z^0$ (+1)	10^{-18} m (+1)	10^{-5} (+1)	10^{-8} (+1)
EM	photon (+1)	∞ (+1)	α (+1)	10^{-20} (+1)
gravity	graviton (+1)	∞ (+1)	10^{-38} (+1)	∞

II.

Particle Type			
Proton	A, B, C, F	+2pt	
Pion	A, D, E	+2pt	
Positron	C, G, H	+2pt	
Neutrino	C, H	+2pt	
J/ψ	A, D, E	+2pt	

A = hadron
B = baryon
C = fermion
D = boson
E = meson
F = nucleon
G = antiparticle
H = lepton

III. a. Estimate ρ_N given $R = 1.2\times10^{-15}$ m

$$\rho = \frac{m}{V} = \frac{1.66\times10^{-27}\,kg}{\frac{4}{3}\pi(1.2\times10^{-15}m)^3} = \frac{1.66\times10^{-27}\,kg}{7.22\times10^{-45}\,m^3} = 2.3\times10^{17}\,\frac{kg}{m^3}$$

$$= 2.3\times10^{14}\,\frac{g}{cm^3}$$

+4pt

III. b. KE_{max} for e^- $\quad n \to p + e^- + \bar{\nu}_e$

$$~~KE_{max}~~ = Q = m_n c^2 - m_p c^2 - m_e c^2 - \cancel{m_{\bar{\nu}_e} c^2}^{\,0}$$

$$= 939.565\text{ MeV} - 938.272\text{ MeV} - 0.511\text{ MeV}$$

$$= 0.782\text{ MeV}$$

III. b (cont)

$$T_{e^-}(max) = \frac{m_p}{m_p + m_e} \cdot Q$$

$$= \frac{938.272\ MeV/c^2}{(938.272\ MeV/c^2 + 0.511\ MeV/c^2)} \quad (0.782\ MeV)$$

$$= 0.781\ MeV \quad +3pt$$

III. c. $n \rightarrow p + e^- + \bar{\nu}_e$

	Reactant	Products	
#baryons	1	1 + 0 + 0 = 1 ✓	+1pt
#leptons	0	0 + 1 + (−1) = 0 ✓	+1pt
charge	0	+1 + (−1) + (0) = 0 ✓	+1pt

IV.

+1pt each

Baryon	Quark content	
n	udd (s=0)	u = up
p	uud (s=0)	d = down
Σ^-	dds (s=−1)	s = strange
Σ^0, Λ_8	uds (s=1)	
Σ^+	uus (s=−1)	
Ξ^-	dss (s=−2)	
Ξ^+	uss (s=−2)	

V Construct the quark wave function of η_8 by requiring it to be normalized and orthogonal to those of π^0 and η_0 given:

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\{|u\bar{u}\rangle - |d\bar{d}\rangle\}$$

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}\{|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\}$$

linear combination determined from orthogonality

if $|\pi^0\rangle \propto \{|u\bar{u}\rangle - |d\bar{d}\rangle\}$, then $\eta_8 \propto \{|u\bar{u}\rangle + |d\bar{d}\rangle\}$

if $|\eta_0\rangle \propto \{|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\}$, then η_8 must also include $-2|s\bar{s}\rangle$

$\therefore$ $|\eta_8\rangle \propto \{|u\bar{u}\rangle + |d\bar{d}\rangle + 2|s\bar{s}\rangle\}$ +5pts

normalization : $\sqrt{(1)^2 + (1)^2 + (-2)^2} = \sqrt{6}$ 5pts

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}\{|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle\}$$

VI a show that $|p\rangle$, $|n\rangle$ are eigenfunctions of τ_3

$$\tau_3|p\rangle = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} = |p\rangle \quad \text{3pts}$$

$$\tau|n\rangle = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = |n\rangle \quad \text{3pts}$$

NUCL Exam 11/28/12 Key

VI b. Construct quark wavefunction for π^- by applying τ_- to π^0

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\{|u\bar{u}\rangle - |d\bar{d}\rangle \; ; \; \cancel{\tau_-|t,t_0\rangle = \sqrt{t(t+1)}}$$

$$\tau_-|t,t_0\rangle = \sqrt{t(t+1) - t_0(t_0-1)}\;|t,t_0-1\rangle$$

$$\tau_-|\pi^0\rangle = \tau_-|u\bar{u}\rangle - \tau_-|d\bar{d}\rangle \quad +6\text{pts}$$

$$\tau_-|u\rangle, \; t=\tfrac{1}{2}, t_0=\tfrac{1}{2} = \sqrt{\tfrac{1}{2}(\tfrac{3}{2}) - \tfrac{1}{2}(-\tfrac{1}{2})}\;|d\rangle \quad +2\text{pts}$$
$$= 1|d\rangle = |d\rangle$$

$$\tau_-|\bar{u}\rangle, \; t=\tfrac{1}{2}, t_0=-\tfrac{1}{2} = \sqrt{\tfrac{1}{2}(\tfrac{3}{2}) + \tfrac{1}{2}(-\tfrac{3}{2})}\;|d\rangle = \phi \quad +2\text{pts}$$

$$\tau_-|d\rangle, \; t=\tfrac{1}{2}, t_0=-\tfrac{1}{2} = \sqrt{\tfrac{1}{2}(\tfrac{3}{2}) + \tfrac{1}{2}(-\tfrac{3}{2})}\;|u\rangle = \phi \quad +2\text{pts}$$

$$\tau_-|\bar{d}\rangle, \; t=\tfrac{1}{2} \; t_0=\tfrac{1}{2} = \sqrt{\tfrac{1}{2}(\tfrac{3}{2}) - \tfrac{1}{2}(-\tfrac{1}{2})}\;|-\bar{u}\rangle \quad +2\text{pts}$$
$$= -\bar{u}\rangle$$

$$\tau_-|\pi^0\rangle = |d\bar{u}\rangle = |\pi^-\rangle$$

VII a. Why does n has instrinsic spin magnetic moment?

+4pts composed of udd quarks, each with magnetic moment due to spin: $\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$

~~VII b. Show that $|\mu_g| = |\mu_g|$~~