# Nuclear Chemistry Cumulative Examination
# November 28, 2012

This exam focuses on nuclear science at the subatomic scale. The exam will be graded out of 100 points, with the distribution indicated at the start of each question. The following table may prove helpful in addressing questions on the exam.

| Quark Flavor | Charge (e) | Spin (s) | Mass (GeV/$c^2$) | t | $t_0$ |
|---|---|---|---|---|---|
| Up (u) | +2/3 | 1/2 | 0.0024 | 1/2 | +1/2 |
| Down (d) | -1/3 | 1/2 | 0.0048 | 1/2 | -1/2 |
| Strange (s) | -1/3 | 1/2 | 0.104 | 0 | 0 |
| Charm (c) | +2/3 | 1/2 | 1.27 | 0 | 0 |
| Beauty (b) | -1/3 | 1/2 | 4.2 | 0 | 0 |
| Top (t) | +2/3 | 1/2 | 171.2 | 0 | 0 |

Some useful constants:

$c = 2.998 \times 10^{8}\ m/s$ $\quad h = 6.626 \times 10^{-34}\ Js$ $\quad e = 1.602 \times 10^{-19}\ C$

$m_p = 1.6276 \times 10^{-27}\ kg = 938.272\ MeV/c^2$ $\quad \alpha = 1/137 = e^2/4\pi\varepsilon_0\hbar c$

$m_n = 1.6749 \times 10^{-27}\ kg = 939.565\ MeV/c^2$ $\quad \varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$

$m_e = 9.1094 \times 10^{-31}\ kg = 0.511\ MeV/c^2$ $\quad \mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$

$k = 1.381 \times 10^{-23}\ JK^{-1}$ $\quad \mu_N = 3.152 \times 10^{-14}\ MeVT^{-1}$ $\quad \mu_B = 5.788 \times 10^{-11}\ MeVT^{-1}$

I. (14 points) *Fundamental Forces*

Copy the following table into your answer book and complete the empty cells

| Force | Exchange boson | Range (m) | Relative Strength | Typical Time Scale (s) |
|---|---|---|---|---|
| Strong | | | 1 | |
| Weak | | | | |
| Electromagnetic | | | | |
| Gravity | | | | |

II. (10 points) *Particle Types*

The following descriptors that can be used to describe subatomic particles: A. hadron; B. baryon; C. fermion; D. boson; E. meson; F. nucleon; G. antiparticle; H. lepton.

Indicate ALL the descriptors that can be applied to the particles listed below:

a. proton
b. pion
c. positron
d. neutrino
e. $J/\Psi$

III. (10 points) *General Properties of Nuclei*

a. Given that the radius of a nucleon inside a nucleus is R = 1.2x10$^{-15}$ m, estimate the density of nuclear matter.

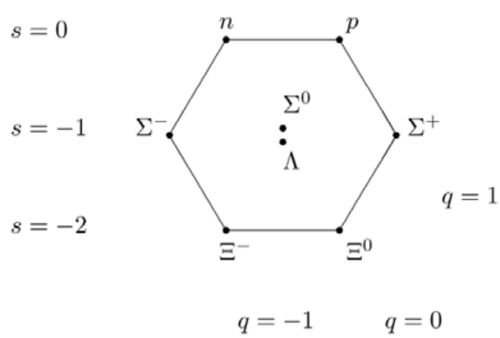
b. Use conservation of momentum and energy to calculate the maximum kinetic energies for electrons released in the decay of the free neutron:

$$n \rightarrow p + e^- + \overline{\nu}_e$$

c. Show that the reaction in III.b satisfies conservation of lepton and baryon numbers.

IV. (8 points) *Quark Composition of Baryons*

The lower-energy baryon octet composed of u-, d-, and s-quarks is given below. Give the quark composition of each baryon.




V. (10 points) *Hadron Wave Functions*

The meson $\eta_8$ is a neutral, isospin-singlet ($t = 0$) particle made of a linear combination of quark-antiquark pairs taken from u-, d-, and s-quarks and their antiparticles. Construct the quark wave function of $\eta_8$ by requiring it to be normalized and orthogonal to those of $\pi^0$ and $\eta_0$ given below:

$$\left|\pi^0\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|u\overline{u}\right\rangle - \left|d\overline{d}\right\rangle\right\}$$

$$\left|\eta_0\right\rangle = \frac{1}{\sqrt{3}}\left\{\left|u\overline{u}\right\rangle + \left|d\overline{d}\right\rangle + \left|s\overline{s}\right\rangle\right\}$$

VI. (20 points) *Isospin*

a. The isospin wave functions for nucleons may be written as:

$$\left|p\right\rangle = \left|t = 1/2, t_0 = +1/2\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_t$$

$$\left|n\right\rangle = \left|t = 1/2, t_0 = -1/2\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_t$$

show that these wave functions are eigenfunctions of the isospin operator $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

b. The isospin lowering operator is given as:

$\tau_{-}\left|t,t_0\right\rangle = \sqrt{t(t+1)-t_0(t_0-1)}\left|t,t_0-1\right\rangle$.

Construct the quark wave function for the $\pi^-$ by applying this relation to the wave function of the $\pi^0$, given by $\left|\pi^0\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|u\bar{u}\right\rangle - \left|d\bar{d}\right\rangle\right\}$.

VII. (10 points) *Quark Magnetic Moments*

Quarks have both spin and charge, therefore they have an intrinsic dipole moment represented in terms of operators as $\boldsymbol{\mu} = g\mathbf{s}\mu_D$, where g is the gyromagnetic ratio, and $\mu_D$ is identical in form to the nuclear magneton except the quark charge and quark mass are considered.

a. Explain the appearance of an intrinsic magnetic moment of the neutron [ $\mu(n) = -1.91\ \mu_N$ ] given the fact that the particle has no net charge.

b. Use the quark structure of the vector mesons $\rho^+$ and $\rho^-$ to show that their magnetic dipole moments are equal in magnitude and opposite in sign.

$\left|\rho^+\right\rangle = \left|t=1, t_0=+1\right\rangle$

$\left|\rho^-\right\rangle = \left|t=1, t_0=-1\right\rangle$

VIII. (10 points) Sketch the Feynman diagram for the process of $\beta^-$ decay outlined below:

$n \rightarrow p + e^- + \bar{\nu}_e$

IX. (8 points) *Subatomic Science in the News*

a. The Large Hadron Collider recently realized an important science milestone, generating world-wide excitement into the possibilities of particle physics discovery.

i. Where is the LHC located?

ii. What particles are accelerated in the machine?

iii. What is the major science objective of the LHC?

b. The 2012 Nobel Prize in Physics was awarded to two scientists for their discoveries that may ultimately lead to “quantum computers”. Provide a few-sentence summary of their seminal work.