This exam will focus on electromagnetic transitions. Show all work when appropriate. A table of possibly useful physical quantities is provided at the end of this exam. The exam is graded out of 100 points with the point breakdown indicated next to each question.

1. (10 points) Consider the transition of a single proton from an initial excited state to the ground state of a particular nucleus which involves a change in the distribution of protons within the nucleus. Schematically describe the difference between electric and magnetic transitions.

   Electric transitions arise from a shift in the distribution of charge in the nucleus and magnetic transitions arise from a shift in the current distribution.

2. (10 points) Estimate the nuclear recoil resulting from the emission of a-233 keV gamma ray from ^{133m}Xe. Remember that amu = 931.5 MeV/c^2.

   Nuclear recoil = $T_r = E_\gamma^2/2M_oc^2 = (0.233\ \text{MeV})^2 / 2(132.906\ \text{amu} * 931.5\ \text{MeV}/c^2/\text{amu} * c^2)$
   $T_r = 2.2*10^{-7}$ MeV = 0.22 eV

3. (10 points) The ground state of ^{72}Ge is 0^+. The first two excited states are 0^+ and 2^+ at energies of 691 and 834 keV, respectively. The transition between the first excited 0^+ state and the ground state is forbidden to occur through gamma-ray emission.
   a. Why is the transition from the first excited state to the ground state forbidden?

      An E0 gamma-ray transition is forbidden on the basis of angular momentum transfer

   b. What is the de-excitation mechanism of the 691 keV state?

      The 691 keV state will decay through internal conversion.

4. (10 points) Reproduce the table below into your blue book and complete the third and fourth columns. The third column represents the change in orbital angular momentum (l) for each transition and the fourth column indicates whether parity change occurs or not.

Radiation Type	Name	Δl	Δparity
E1	Electric dipole	1	Yes
M1	Magnetic dipole	1	No
E2	Electric quadrupole	2	No
M2	Magnetic quadrupole	2	Yes
E3	Electric octopole	3	Yes

5. (15 points) ^{135}Xe has an J^{π} = 11/2$^-$ isomeric state at 526.6 keV which decays directly to the ground state with a half-life of $t_{1/2}$ = 15.3 minutes. A pure isomeric sample was obtained and the decay of the 11/2$^-$ state was monitored. A total of 10,000 526-keV gamma rays, 1950 492-keV conversion electrons, and 336 conversion electrons in a group of unresolved transitions at 521 keV, all corrected for detection efficiency, were observed and attributed to the decay of the isomeric state.

   a. What is the internal K-conversion coefficient?

      α_K = K conversion electrons / gamma-rays = 1950 / 10000 = 0.195

   b. What is the multipolarity of the isomeric transition based on predicted conversion coefficients presented in Fig. 1 at the end of this exam?

      M4

   c. What is the spin and parity of the ^{135}Xe ground state?

      3/2$^+$

   d. What is the binding energy of the K-shell electron?

      K-shell binding energy = (526 keV – 492 keV) = 34 keV

   e. Suggest the origin of the unresolved transitions at 521 keV.

      L shell electron conversion

6. (10 points) Electromagnetic transition rates are directly proportional to the reduced transition probability, $B(J^{\pi}_i \rightarrow J^{\pi}_f)$ according to the following equation:

$$\lambda\left(\ell, J_i^{\pi} \rightarrow J_f^{\pi}\right) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{k^{2\ell+1}}{\hbar} B\left(\ell, J_i^{\pi} \rightarrow J_f^{\pi}\right)$$

The reduced transition probability $B(J^{\pi}_i \rightarrow J^{\pi}_f)$ is the matrix element for the multipole operator, O(L), (either electromagnetic or magnetic):

$$B\left(\ell, J_i^{\pi} \rightarrow J_f^{\pi}\right) = \left|< J_i \right| O(\ell M) \left| J_f \right| >^2$$

A simple estimate for transition rates was derived by Weisskopf and provides a guide for evaluating the nature of electromagnetic transitions. Considering electric transitions and starting with the multipole operator of the form $er^L Y_{LM}(\theta,\phi)$ where L is the multipole order and Y_{LM} are the standard spherical harmonics (the derivation for electromagnetic transitions is shown but a similar derivation can be applied for the magnetic transitions)

$$B\left(\ell, J_i^{\pi} \rightarrow J_f^{\pi}\right) = \left|< J_i \right| e r^{\ell} Y_{\ell M}(\theta,\phi) \left| J_f \right| >^2$$

Evaluating the radial portion of the integral as

$$\frac{\int_0^R r^2 r^\ell dr}{\int_0^R r^2 dr} = \frac{3}{3+\ell} R^\ell$$

where R is the radius of the nucleus and replacing the angular integral with 1/4p the reduced transition probability becomes (for electric transitions)

$$B\,(E\ell) = \frac{1}{4\pi}\left[\frac{3}{3+\ell}\right]^2 R^{2\ell} e^2 fm^{2\ell}$$

resulting in

$$\lambda\left(E\ell, J_i^\pi \rightarrow J_f^\pi\right) = \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{k^{2\ell+1}}{\hbar} \frac{e^2}{4\pi}\left[\frac{3}{3+\ell}\right]^2 R^{2\ell}$$

Using R = $R_0A^{1/3}$ the Weisskopf estimate for some of the electric transitions become

$$\lambda\,(E1) = 1.0 \; x \; 10^{14} A^{2/3} E_\gamma^3$$

$$\lambda\,(E2) = 7.3 \; x \; 10^{7} A^{\frac{4}{3}} E_\gamma^5$$

$$\lambda\,(E3) = 3.4 \; x \; 10^{1} A^{2} E_\gamma^7$$

$$\lambda\,(E4) = 1.1 \; x \; 10^{-5} A^{\frac{8}{3}} E_\gamma^9$$

Where λ is in s^{-1}, A is the mass number of the nucleus and E_γ is the gamma-ray energy in MeV. Describe in words, with equations if necessary, the assumptions made to derive the Weisskopf estimates.

Weisskopf estimates assume that the electromagnetic transition is due to the transition of a single nucleon from an initial state to a final state, the nucleus can be approximated as a sphere of constant density, and the radius of the nucleus is $R_oA^{1/3}$ with $R_o \approx 1.2$ fm.

7. (10 points) The beta decay of ^{135}Xe predominately proceeds through a transition to the first excited state in ^{135}Cs at 249.8 keV. The spin and parity of the ^{135}Cs first excited state and ground state are $5/2^+$ and $7/2^+$ respectively.
   a. List all possible multipolarities that could contribute to the gamma-ray transition between the first excited state and the ground state in ^{135}Cs.

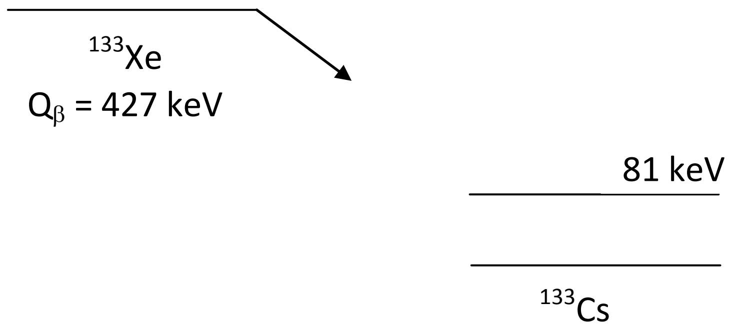
      M1, E2, M3, E4, M5, E6

   b. List the likely mulitpolarities that will contribute to the gamma-ray transition between the first excited state and the ground state.

      M1 + E2

8. (30 points) The nucleus ^{133}Xe can be used as a monitor of clandestine nuclear weapons tests. Detecting the presence of ^{133}Xe in an atmospheric air sample is accomplished by monitoring either the IT decay of an isomeric state in ^{133}Xe or through the gamma rays emitted following the beta decay of ^{133}Xe to ^{133}Cs. Reproduce the following figure in your blue book.

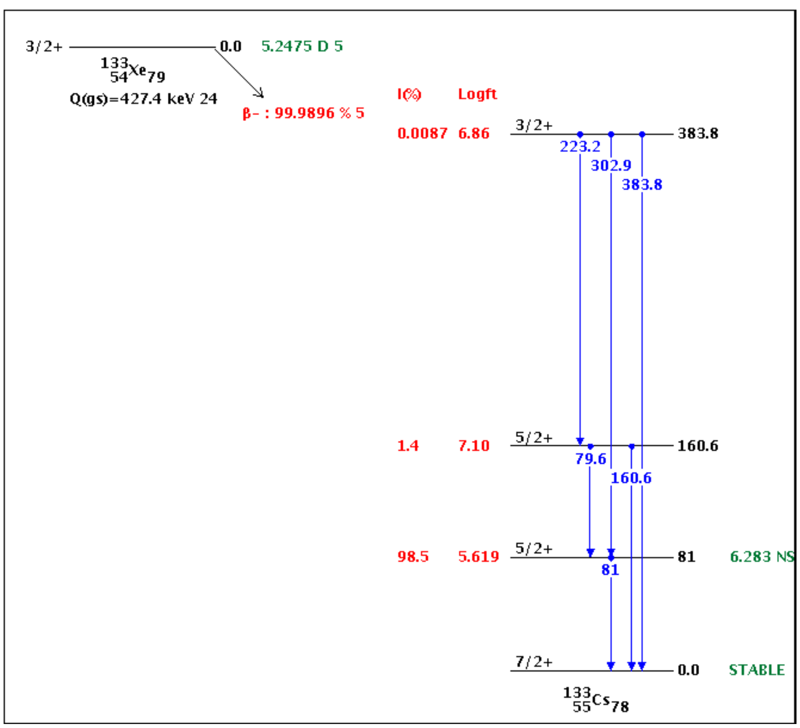
$3/2^+$



Based on the data provided in the table below construct the level scheme of ^{133}Cs populated in the beta-decay of ^{133}Xe. Indicate the energy and spin and parity of each level.

Transition energy (keV)	Comments
79.6	In coincidence with 81.0 and 223.2 keV transitions
81.0	$t_{1/2}$ = 6.283 ns, populates a 3/2+ transition
160.6	M1+E2
223.2	M1 + E2, in coincidence with 79.6, 81.0, and 160.6, keV transitions
302.8	in coincidence with 81.0-keV transition
383.8	α_K = 0.017

From the NNDC website:



9. (10 points) Assignment of spins and parities to individual levels in a nucleus is greatly aided by a measurement of a gamma-ray angular distribution. In typically laboratory settings, the various

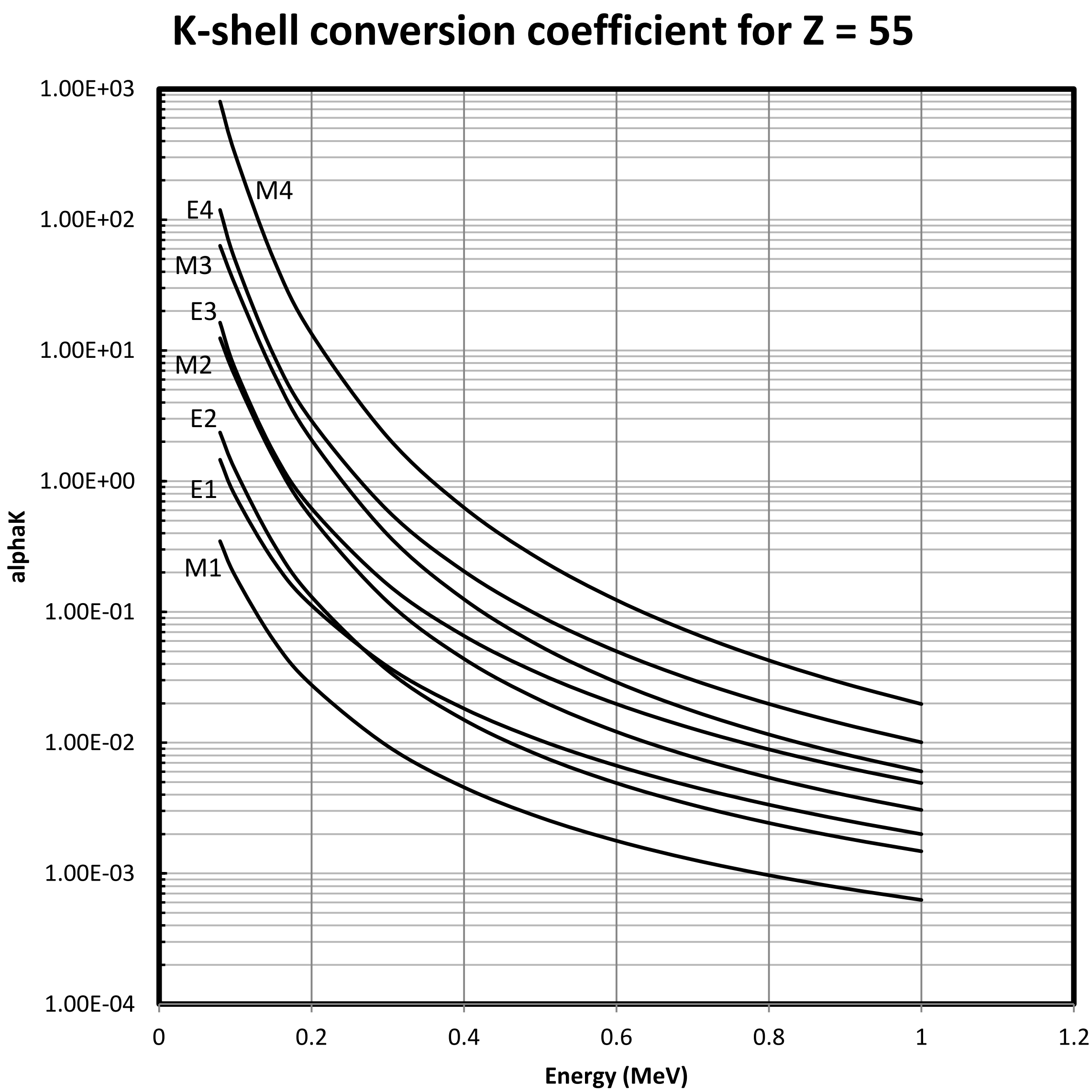
m_i substates of a nuclear level with total spin and parity J^π are equally populated resulting in an isotropic emission of gamma-rays from the level. Suggest two mechanisms for creating an unequal population of the m_i substates.

Unequal populations of m_i substates can be obtained by either (1) cooling the a sample in the presence of a strong magnetic field or (2) detecting a preceding gamma-ray decay and perform an angular correlation experiment.

Useful Data

Multipolarity	Transition rate (s^{-1})	Multipolarity	Transition rate (s^{-1})
E1	$1.0\ x\ 10^{14} A^{2/3} E_\gamma^3$	M1	$3.2\ x\ 10^{13} E_\gamma^3$
E2	$7.3\ x\ 10^{7} A^{\frac{4}{3}} E_\gamma^5$	M2	$2.2\ x\ 10^{7} A^{\frac{2}{3}} E_\gamma^5$
E3	$3.4\ x\ 10^{1} A^{2} E_\gamma^7$	M3	$1.0\ x\ 10^{1} A^{4/3} E_\gamma^7$
E4	$1.1\ x\ 10^{-5} A^{\frac{8}{3}} E_\gamma^9$	M4	$3.8\ x\ 10^{-6} A^{2} E_\gamma^9$

Isotope	Mass (amu)
130Xe	129.903
131Xe	130.905
132Xe	131.904
133Xe	132.906
134Xe	133.905
135Xe	134.907


K-shell conversion coefficient for Z = 55
1.00E+03
1.00E+02
1.00E+01
1.00E+00
1.00E-01
1.00E-02
1.00E-03
1.00E-04
alphaK
M4
E4
M3
E3
M2
E2
E1
M1
0
0.2
0.4
0.6
0.8
1
1.2
Energy (MeV)