Nuclear Chemistry Cumulative Examination
October 19, 2011

This exam focuses on the magnetic and electric properties of nuclei. The exam will be graded out of 100 points, with the distribution indicated at the start of each question.

I. *Magnetic dipole moment of the proton and neutron*

a. (10 points). The classical treatment of the magnetic dipole moment considers a circular loop carrying current i and enclosing an area A produces $\mu = iA$. **Show** that the rotation of a nuclear charge e (1.602×10^{-19} C), moving with velocity v in a circle of radius r can lead to the working definition of the nuclear magnetic moment

$$\mu = \frac{e\hbar}{2m}\ell \quad \text{(I.1)}$$

where m is the mass of the proton (1.672×10^{-27} kg), ℓ is the angular momentum quantum number of the orbit, and $\hbar = h/2\pi = 1.055 \times 10^{-34}$ Js.

b. (10 points). The neutron has no net electric charge, yet has a non-zero magnetic moment [μ(n) = −1.91 μ_N]. **Explain** this apparent discrepancy with the relation given in equation (I.1).

c. (10 points). Equation (I.1) can be rewritten in a more useful form for a single nucleon:

$$\mu = g_\ell \ell \mu_N + g_s s \mu_N \quad \text{(I.2)}$$

where both the orbital (ℓ) and spin (s) contributions are now considered and

$$\mu_N = \frac{e\hbar}{2m} \quad \text{(I.3)}$$

is the nuclear magneton (5.050×10^{-27} J/T). For spin-1/2 point particles, the Dirac equation gives $g_s = 2$. The g_s value for the free electron is 2.0023, close to the Dirac expectation. However, the value of g_s for the free proton is 5.5856 and for the free neutron $g_s = -3.826$. **Justify** the deviations from Dirac theory of the proton and neutron free g-factors.

II. *Magnetic dipole moments of multi-nucleon systems*

a. (10 points) The deuteron (^2H) consists of a single proton and single neutron coupled to total angular momentum $I = 1$ in the lowest energy configuration (the ground state). **Estimate** the magnetic dipole moment of ^2H using information provided in question I.c,.

b. (10 points) The ground-state wavefunction of ^2H is believed to contain a small admixture of d state ($\ell = 2$) along with the pure $\ell = 0$ component as calculated in question II.a. **Give expressions** for both the wavefunction and magnetic moment of ^2H that include the amplitudes a_s and a_d of the s- and d-state contributions.

c. (10 points) **Describe** a method to measure the magnetic dipole moment of ^2H.

d. (10 points) The nucleus ^{20}Ne contains 10 protons and 10 neutrons. **Explain** why the magnet dipole moment of the ground state of this nucleus is zero.

Over →

III. *Electric quadrupole moments of multi-nucleon systems*

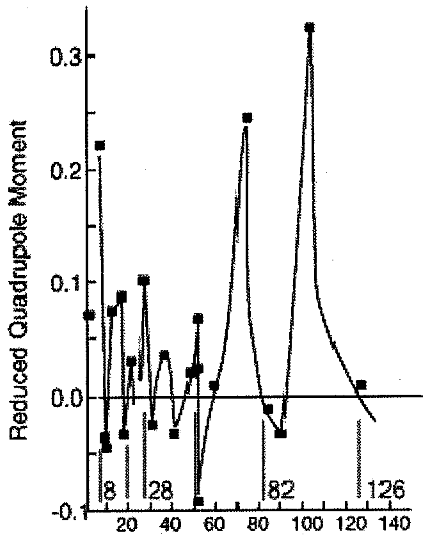
The electric quadrupole moment of a nucleus can be estimated assuming the nucleus has a spheroid shape generated by rotating an ellipse about one of its axes. The resulting relation is

$$Q = \frac{2}{5} Ze(a^2 - c^2), \quad \text{(III.1)}$$

where Ze is the nuclear charge, and c and a are the semi-minor and -major axes, respectively.

a. (10 points) For ^{181}Ta, Q/e = +4.20 barns. **Calculate** the ratio of the semi-major to semi-minor axes of this nucleus. Note that 1 barn = 10^{-24} cm^2 and $R^2 = 0.5(a^2+c^2) = 1.2\ \text{fm}^2 A^{2/3}$.

b. (10 points) In the figure below, the ground state electric quadrupole moments of selected odd-mass nuclides are plotted as a function of the number of odd nucleons (protons or neutrons) in the system. **Explain** why, for nuclei with certain nucleon numbers (8, 20, 28, 50, 82, 126), that Q ~ 0.



IV. *Electric dipole moment of the neutron*

The search for an electric dipole moment (EDM) of the neutron is one of the exciting possibilities for finding physics beyond the current standard model. The current experimental limits are all consistent with zero, in agreement with the predictions of the standard model.

a. (5 points) **Estimate** for the magnitude of the EDM of the neutron, and **justify** all assumptions made in arriving at the estimate.

b. (5 points) A neutron EDM larger than standard model predictions would have serious implications on CPT symmetry, which is believed to be exact. **Define** the acronyms C, P, and T, and **indicate** which of the three (C, P, or T) is violated by a non-zero neutron EDM.

Ia. $\mu = i A$

if current caused by a charge e, moving with speed v in a circle of radius r, then

period $= 2\pi r/v$

$$i = \frac{e}{(2\pi r/v)} \qquad A = \pi r^2$$

$$\mu = \frac{e}{(2\pi r/v)} \pi r^2 = \frac{evr}{2} \cdot \frac{m}{m} = \frac{e \cdot rmv}{2m} = \frac{eL}{2m}$$

from classical to quantum mechanical

$$L = \ell \hbar$$

$$\mu = \frac{e\hbar}{2m} \ell$$

Ib. Neutron is composed of up and down quarks (udd)
Quarks carry charge $u = +2/3\, e$
$d = -1/3\, e$
the charged nature of sub-nucleonic particles produce a non-zero μ for neutron

Ic. Again, both protons (uud) and neutrons (udd) are composed of quarks. Since nuclions are not point particles, they deviate significantly from the Dirac expectation.

Ia. $^{2}_{1}H_{1}$ 1 proton
1 neutron

naively, would assume that μ is combination of spin g factors of proton and neutron

$$\mu = \frac{1}{2}(g_s(p) + g_s(n))$$
$$= \frac{1}{2}(+5.8856 + (-3.826))$$
$$= +\frac{1}{2}(1.7596) = \boxed{0.8798\,\mu_N}$$

Ib.

$$\psi = a_s \psi_{\ell=0} + a_d \psi_{\ell=2}$$

$$\mu = a_s^2 \mu_{\ell=0} + a_d^2 \mu_{\ell=2}$$

IIc. ^{2}H measured by molecular beam magnetic resonance method. See Kellogg, Rabi, Ramsey and Zacharias PR 57, 677 (1940)

static magnetic field } search for absorption of
varying rf field } rf photons

IId. $^{20}_{10}Ne_{10}$. magnetic moment is zero since all protons and all neutrons couple to $J=0$ in the ground state. (strong p-p and n-n pairing)

III a. $^{181}_{73}Ta$ $\frac{Q}{e} = +4.20$ barns

what are a + c   2 equations

$$\frac{Q}{e} = \frac{2}{5} Z (a^2 - c^2)$$

$$R^2 = \frac{1}{2}(a^2 + \cancel{b}^2) = 1.2\,fm^2 A^{2/3}$$

$$\frac{4.2\times10^{-24}\,cm^2}{73}\left(\frac{5}{2}\right) = (a^2-c^2) = 1.438\times10^{-25}\,cm^2 \times\left(\frac{10^{-2}\,m}{cm}\right)^2 = 1.438\times10^{-29}\,m^2$$

$$2\cdot1.2\times10^{-30}\,m^2\,(181)^{2/3} = (a^2+\cancel{b}^2) = 7.679\times10^{-29}\,m^2$$

$$a^2 - c^2 = 1.438\times10^{-29}\,m^2 \qquad a^2 = c^2 + 1.438\times10^{-29}\,m^2$$

$$a^2 + c^2 = 7.679\times10^{-29}\,m^2 \qquad = 2c^2 + 1.438\times10^{-29}\,m^2 = 7.679\times10^{-29}\,m^2$$

$$2c^2 = 6.241\times10^{-29}\,m^2$$

$$c^2 = 3.1205\times10^{-29}\,m^2$$

$$c = 5.58\times10^{-15}\,m$$

$$a^2 + (5.58\times10^{-15}\,m)^2 = 7.679\times10^{-29}\,m^2$$

$$a^2 + 3.1205\times10^{-29}\,m^2 = 7.679\times10^{-29}\,m^2$$

$$a^2 = 4.5585\times10^{-29}\,m^2$$

$$a = 6.751\times10^{-15}\,m$$

$$\frac{a}{c} = \frac{6.751\times10^{-15}\,m}{5.58\times10^{-15}\,m} = \boxed{1.21}$$

III b. For the numbers 8, 20, 28, 50, 82, 126, there is a large gap in single-~~neutr~~ nucleon shell structure. These "magic" nucleon numbers show extra "stability". Therefore, they are spherical in shape and should have $Q \sim 0$

Nuclear Key 10/19/2011

IV a.

u
d d
neutron

size ~ 1×10^{-15} m (1 fm)
charges $u = +\frac{2}{3}e$
$d = -\frac{1}{3}e$ (each)

if full separation of u and d quarks in nucleus, then $d = q z$ q is charge, z is separation distance

$$d = \left[\frac{2}{3} - \left(-\frac{2}{3}\right)\right] e \, (1 \times 10^{-13}\text{ cm})$$
$$= \frac{4}{3} e \, (1 \times 10^{-13}\text{ cm}) = \boxed{1.33 \times 10^{-13}\ e\,\text{cm}}$$

(actual size is $< 10^{-26}$ e cm)

IV b. C = charge conjugation symmetry

P = parity change symmetry

T = time reversal symmetry.

CP broken in decay of neutral mesons.

∴ T is expected to be broken, if CPT is an exact symmetry.

evidence for neutron EDM would violate T symmetry.