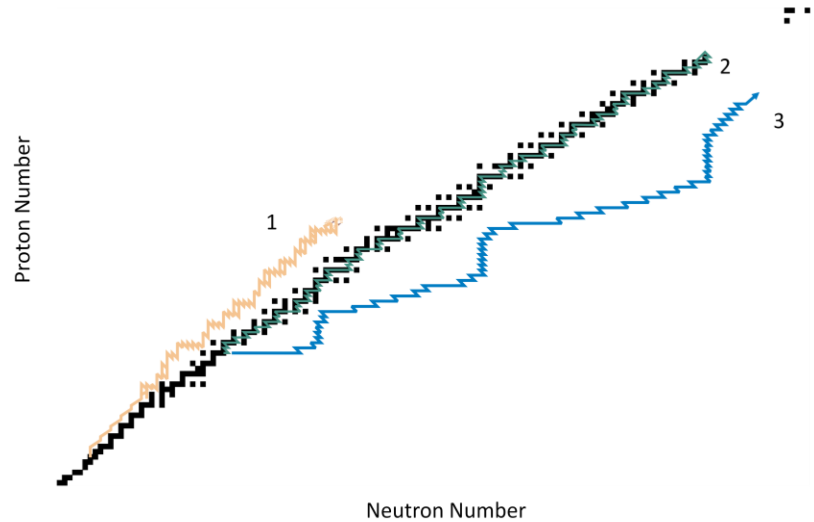
Nuclear Chemistry Cumulative Exam – Wednesday, Feb 16, 2011

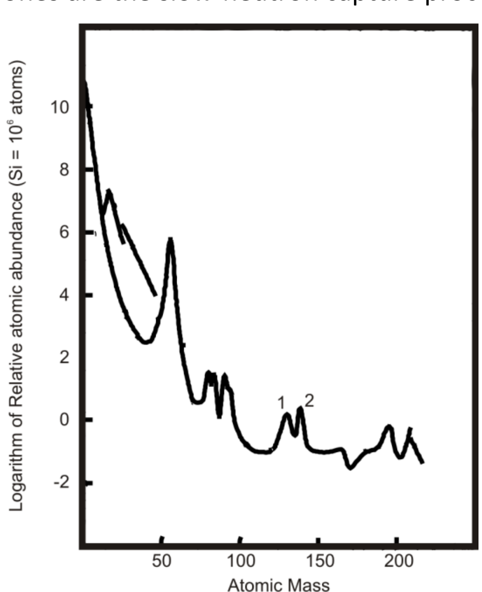
This examination is concerned with nuclear reactions in stellar environments as discussed in the textbook Modern Nuclear Chemistry by Loveland, Morrissey, and Seaborg. Possibly useful equations and constants are provided on page 4. There are a total of 100 points on this exam.

1. (5 pts) Shown below is the chart of the nuclides with approximate paths for three different astrophysical processes. Label each numbered path with the name of the process responsible, choosing from either s-process, rp-process, or r-process.
    a. s-process
    b. rp-process
    c. r-process



1 – rp-process
2 – s-process
3 – r-process

2. (15 pts) Shown below is a schematic diagram of the relative solar abundances of the elements as a function of atomic mass. Heavy elements can be created in multiple processes. Two common ones are the slow-neutron capture process, characterized by neutron capture time scales that are long compared to beta decay lifetimes and the rapid-neutron capture process in which neutron capture time scales are shorter than beta decay lifetimes.



Identify the peaks labeled 1 and 2 as originating from either the s-process or r-process and explain why peak 1 is at a lower mass compared to peak 2.

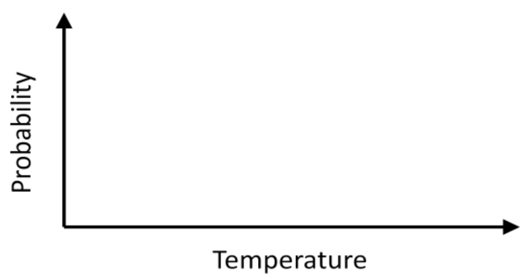
Peak 1 is due to the r-process and peak 2 is due to the s-process. The r-process path is very far from stability and reaches the magic numbers at lower values of Z resulting in a lower mass peak as compared to the s-process

3. (25 pts) The temperature averaged reaction rate per particle pair $<\sigma v>$ is given by the equation

$$<\sigma v> = (\frac{8}{\pi\mu})^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty S(E)\exp\left(-\frac{E}{kT}-\frac{b}{E^{\frac{1}{2}}}\right)dE$$

where μ is the reduced mass, k is Boltzmann's constant, T is the gas temperature, S(E) is the astrophysical S-factor, and b is $0.989Z_1Z_2\mu^{1/2}(\mathrm{MeV})^{1/2}$. The integral is dominated by the exponential term and represents the overlap between the Maxwell-Boltzmann distribution and a sub-barrier tunneling probability called the Gamow factor. For the following questions consider the simple non-resonant fusion reaction between two protons in a stellar environment.

a. Estimate the Coulomb barrier for this reaction
b. Reproduce the following figure in your exam book and qualitatively sketch the Maxwell-Boltzmann distribution and Gamow factor. Based on your sketch, indicate the location of the Gamow peak and explain its significance?
c. Show that the maximum of the Gamow peak occurs at $E_o = (bkT/2)^{2/3}$



a. $V_C = \frac{z_1 z_2 e^2}{r_1+r_2} = \frac{1*1*\left(\frac{1}{137.036}\right)*197.327}{(1.3214+1.3214)} = 0.54\ MeV$

b. See Fig. 12.9 on page 344 of <u>Modern Nuclear Chemistry</u>

c. Maximize exponential term: $(-\frac{E}{kT}-\frac{b}{E^{\frac{1}{2}}})$

Derivative : $\left(-\frac{1}{kT}+\frac{b}{2E^{\frac{3}{2}}}\right) = 0$

$$\frac{1}{kT} = \frac{b}{2E^{\frac{3}{2}}}$$

$$E = (\frac{bkT}{2})^{2/3}$$

4. (20 pts) Consider the sequence of three stable Te isotopes; ^{123}Te, ^{124}Te, and ^{125}Te. ^{123}Te and ^{124}Te are only produced from the s-process while the abundance of ^{125}Te is produced through both s- and r-processes. For the following questions assume that the neutron capture cross section is 808 mb, 155 mb, and 431 mb for ^{123}Te, ^{124}Te, and ^{125}Te, respectively and the abundance of ^{124}Te and ^{125}Te are 0.2319 and 0.3437 per 10^6 Si atoms, respectively.
   a. What is the abundance of ^{123}Te in the s-process at equilibrium?
   b. What percentage of ^{125}Te is produced by the r-process?

   For an s-process at equilibrium

$$\frac{dN_a}{dt} = \sigma_{A-1}N_{A-1} - \sigma_A N_A = 0$$

$$N_{123} = \frac{\sigma_{124}N_{124}}{\sigma_{123}} = \frac{155\text{mb} * 0.2319}{808\text{ mb}} = 0.044$$

$$N_{125} = \frac{\sigma_{124}N_{124}}{\sigma_{125}} = \frac{155\text{mb} * 0.2319}{431\text{ mb}} = 0.0834 \text{ from the s} - \text{process}$$

$$\%\,\text{r} - \text{process} = \frac{0.3437 - 0.0834}{0.3437} = 75\%$$

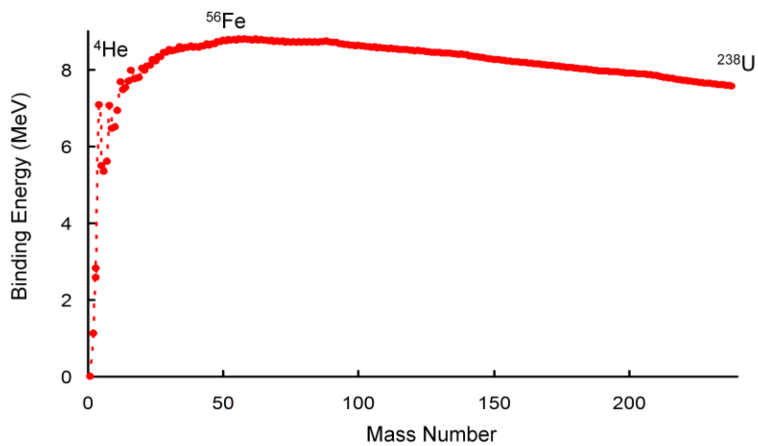
5. (10 pts) There are two different reaction mechanisms for the fusion of two protons into a deuterium nucleus:

   (1) $p + p \rightarrow d + e^+ + \nu_e$
   (2) $p + p + e^- \rightarrow d + \nu_e$

   Explain why reaction (1) results in a broad spectrum of neutrino energies and why reaction (2) is a monoenergetic neutrino source.

   Reaction 1 is a three body final state and the energy of the products is shared between all of them. Reaction 2 is a two-body process leading to a monoenergetic neutrino source.

6. (10 pts) The binding energy per nucleon as a function of mass number is shown below. Based on the figure, explain why fusion processes within a star will not contribute to the creation of elements with masses heavier than approximately A ~ 60.



Fusion reactions producing nuclei with mass greater than ~60 would be endoergic and would not contribute to the creation of heavier elements.

7. (5 pts) The energy produced by the sun is approximately $4*10^{26}$ J/s. Assuming that the energy production is from hydrogen burning within the sun according to the overall reaction

$$4\ p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e \quad Q = 26.7\ MeV$$

Estimate the rate of hydrogen consumption in the sun in kg / second.

$$4 * 10^{26}\frac{\mathrm{J}}{\mathrm{s}} * \frac{1\ \mathrm{eV}}{1.602177 * 10^{-19}\ \mathrm{J}} * \frac{1\ MeV}{1{,}000{,}000\ eV} = 2.50 * 10^{39}\ MeV/s$$

$$2.50 * 10^{39}\frac{MeV}{s} * \frac{1}{26.7\ MeV} = 9.35 * 10^{37}\ reactions/sec$$

$$9.35 * 10^{37}\frac{reactions}{sec} * \frac{4\ p}{reaction} * \frac{1\ amu}{p} * \frac{1.66054 * 10^{-27}\ \mathrm{kg}}{amu} = 6.21 * 10^{11} kg/s$$

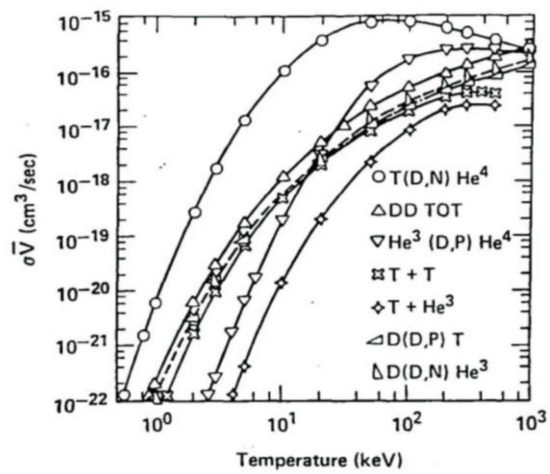
8. (10 pts) The National Ignition Facility is being constructed at Livermore National Laboratory to demonstrate energy gain - the generation of energy from a fusion reaction in excess of the energy required to initiate the reaction. With three isotopes of hydrogen available, H (hydrogen), D (deuterium), and T (tritium) there are multiple possible reactions such as:

$$D + D \rightarrow {}^{3}He + n$$

$$T + T \rightarrow {}^{4}He + 2n$$

$$D + T \rightarrow {}^{4}He + n$$

The reaction rate as a function of temperature for a variety of fusion reactions is show below.



Which of the three reactions D+D, T+T, or D+T has the greatest chance of success at NIF based on both energy generation and reaction rate?

Energy released

D + D → ^{3}He + n;

$$Q = \sum \Delta(\text{reactants}) - \sum \Delta(\text{products}) = 2 * 13.136 - 14.931 - 8.071 = 3.27 \text{ MeV}$$

T +T → ^{4}He + 2n

$$Q = 2 * 14.591 - 2.425 - 2 * 8.071 = 10.6 \text{ MeV}$$

D +T → ^{4}He + n

$$Q = 13.136 + 14.591 - 2.425 - 8.071 = 17.2 \text{ MeV}$$

The D + T reaction will release the most energy and also has the highest reaction rate across all temperatures between 0 and 1000 keV in the figure making it the most likely candidate for demonstrating energy gain at NIF.

$$P(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(\frac{-mv^2}{2kT}\right)$$

$$\sigma(E) = \frac{1}{E} \exp\left[-31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}\right.$$

$$\sigma(E) = \pi\lambda^2 \left[\frac{2J_r + 1}{(2J_x + 1)(2J_y + 1)}\right] \frac{\Gamma_{\text{in}}\Gamma_{\text{out}}}{(E - E_r)^2 + \frac{\Gamma_{\text{tot}}^2}{4}}$$

$$R = N\sigma\phi$$

$$\frac{dN_a}{dt} = \sigma_{A-1} N_{A-1} - \sigma_A N_A$$

$$Q = \sum \Delta(\text{reactants}) - \sum \Delta(\text{products})$$

$$T(K) = \frac{1.5 \text{ x } 10^{10}}{\sqrt{t(s)}}$$

$$V_C = \frac{z_1 z_2 e^2}{r_1 + r_2}$$

Table of Mass Excess, Δ

Nuclide	Δ (MeV)	Nuclide	Δ (MeV)
n	8.071	^{5}Li	11.68
^{1}H	7.289	^{6}Li	14.087
^{2}H	13.136	^{7}Li	14.908
^{3}H	14.590	^{8}Li	20.947
^{3}He	14.931	^{7}Be	15.770
^{4}He	2.425	^{8}Be	4.942
^{5}He	11.39	^{9}Be	11.348
^{6}He	17.595	^{10}Be	12.607

Constants:

Atomic mass unit	931.494 MeV	Atomic mass unit	1.66054*10^{-27} kg
Proton radius	1.3214*10^{-15} m	Neutron radius	1.3196*10^{-15} m
m_e	0.510999 MeV	m_e	9.10939*10^{-27} kg
m_p	938.272 MeV	m_p	1.67262*10^{-27} kg
m_n	939.566 MeV	m_n	1.67493*10^{-27} kg
C	2.99792458*10^{8} m/s	N_a	6.022*10^{23} mol^{-1}
h	6.62607*10^{-34} J/s	k	1.3806*10^{-23} J/K
e^2/hc	137.036	hc	197.327 MeV fm

Conversions:

1 eV = 1.602177*10^{-19} J

1 Ci = 3.700*10^{10} Bq

1 erg = 10^{-7} J