Nuclear Chemistry Cumulative Examination
September 15, 2010

The examination will focus on radioactive decay kinetics. Support all answers with brief justifications where appropriate. A periodic table is available on the wall at the front of the room.

Some helpful relations:

$A = N\lambda$ $T_{1/2} = \ln 2 / \lambda$ $1\ \text{Ci} = 3.7\times10^{10}$ dps $N_t = N_o e^{-\lambda t}$ $N_A = 6.02\times10^{23}\ \text{mol}^{-1}$

Q1 (15 points): Radioactive decay follows first-order kinetics. Sketch the following:

a. Activity as a function of time.
b. Natural logarithm of the activity as a function of time.
c. Activity as a function of the natural logarithm of time.

Q2 (10 points): Determine the number of grams of radioactive atoms contained in 1 mCi of ^{24}Na, a β^- emitter with a half-life of 14.95 h.

Q3 (10 points): The half-life of ^{22}Na is 3.604 y. It decays by the emission of positrons (β^+) with a branching ratio of 89%. The competing decay branch is electron capture (EC), which happens 11% of the time. Calculate the partial decay half-life for each of the modes of decay.

Q4 (20 points): A sample of plutonium consists of ^{239}Pu ($T_{1/2} = 2.41 \times 10^4$ y) and ^{240}Pu ($T_{1/2} = 6.56 \times 10^3$ y) in unknown proportions. The specific activity was found to be 1.72×10^8 disintegrations per minute per milligram. What is the composition of the sample?

Q5 (15 points): Radioactive ^{14}C ($T_{1/2} = 5.72 \times 10^3$ y) produced in the earth's atmosphere is taken up by living things, resulting in an average ^{14}C radioactivity of 10 disintegrations per minute per gram of carbon. Wood taken from an Egyptian tomb had a ^{14}C content of 7.62 disintegrations per minute per gram of carbon.

a. Estimate the age of the wood.
b. List the assumptions made in determining the estimate.

Q6 (10 points): The activity of a radioactive nuclide at the end of an irradiation can be expressed as:

$$A = N\lambda = R\left(1 - e^{-\lambda t}\right)$$

where *R* is the production rate and *t* is the irradiation time. Sketch the activity as a function of irradiation time, and justify why an irradiation time of order 3 times the half-life is sufficient to maximize production of the product nuclide.

The final two questions focus on successive radioactive decays. For a chain of *n* members with the assumption that at $t = 0$ the parent nuclide is solely present (N_1^0), the general solution for the number of atoms of any member of the chain was provided by H. Bateman:

$$N_n = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + \cdots C_n e^{-\lambda_n t}$$

$$C_1 = \frac{\lambda_1 \lambda_2 \cdots \lambda_{n-1}}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)\cdots(\lambda_n - \lambda_1)} N_1^0$$

$$C_2 = \frac{\lambda_1 \lambda_2 \cdots \lambda_{n-1}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\cdots(\lambda_n - \lambda_2)} N_1^0, \text{and so on}$$

Q7 (10 points): Show that the time required for the daughter to reach its maximum activity in a sequential decay is:

$$t_{max} = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2}{\lambda_1}$$

for the case of non-equilibrium decay ($\lambda_1 > \lambda_2$).

Q8 (10 points): Show that for secular equilibrium, where the parent is very much longer lived (> 10^4 times longer) than the daughter, the activities at long times of the parent and daughter are the same: that is, $A_1 = A_2$.