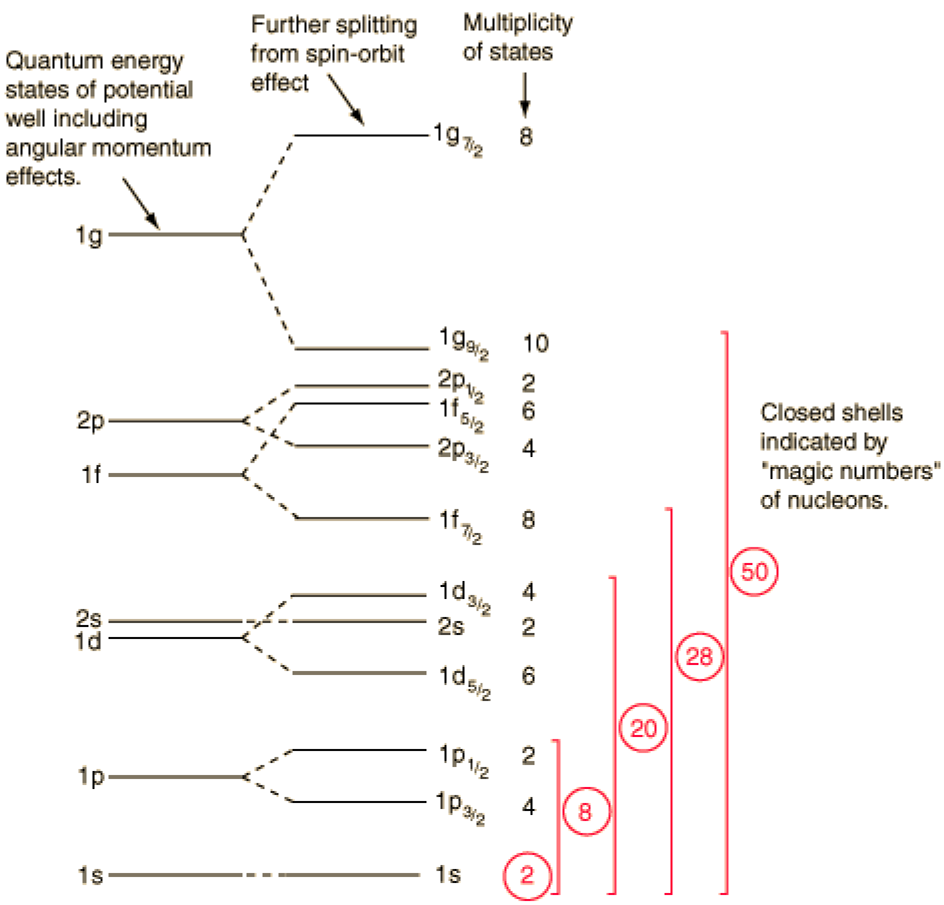
Nuclear Chemistry Cumulative Examination
Wednesday, 17 November 2o1o

This examination is concerned with the general structure of nuclei and simple nuclear shell model as discussed in the textbook Modern Nuclear Chemistry by Loveland, Morrissey & Seaborg.

The exam has a total of 50 points.



1. (20 points) The deuteron, ^{2}H or simply "d", is the simplest nucleus but its properties provide a large amount of information about the nuclear force. It is also the lightest self-mirror nucleus. Answer the following ten questions related to the deuteron and isospin.

   (a) There are only two plausible values for the total angular momentum, j, of the ground state of the deuterium nucleus, what are they? Which value of j corresponds to the ground state configuration for the deuterium nucleus?

S=1/2 for neutron and proton, thus J=0 or J=1, only. J=1 is the ground state. Both nucleon in 1s state, ℓ=0, total L=0, so parity = EVEN or "+"

(b) The nucleus $^{30}P_{15}$ is an odd-odd self-mirror nucleus has a valence proton in the $\pi 2s_{1/2}$ and a valence neutron in the $\nu 2s_{1/2}$ orbitals. What do you expect the spin and parity of the ground state of ^{30}P will be and why?

Single odd particles in s-orbitals, thus, same paring of aligned spins as in deuterium, J=1.

(c) What is the configuration of nucleons (proton and neutron orbital occupancy) in tritium? What is the ground state spin of a tritium nucleus?

π **$1s^1$, ν $1s^2$, neutrons are paired, so J=1/2 from odd proton**

(d) Isospin is a pseudo-quantum number used to describe nuclear states in light nuclei. What are the values of the isospin, T, and its projection, T_z, for the ground state of deuterium?

T_z=(N-Z)/2 = 0, thus T can only be zero.

(e) Identify the three possible isobars for A=2.

^{2}He (aka diproton), ^{2}H (aka deuterium), and dineutron (sometimes written ^{2}n)

(f) Give the value of isospin, T, and its projection, T_z, for the ground state of each of the three A=2 isobars.

^{2}He T=1, T_z=-1; ^{2}H T=0, T_z=0; ^{2}n T=1, T_z=+1

(g) How does the relative stability (for example, mass or binding energy) change in a typical set of three isobars as one goes from neutron deficient to neutron rich?

The total stability generally INCREASES with neutron number due to the disruptive influence of the Coulomb force.

(h) Based on your answers to the previous questions, the deuteron should an have an excited state. Do you expect this state to be bound or unbound? Explain.

This state should have a similar energy to the T=1 states in the isobaric "nuclei" – both of these are unbound, thus, the excited state of the deuteron would be expected to be unbound.

(i) What are the values of the isospin, T, and its projection, T_z, for the excited state of the deuteron?

T=1, T_z=0

(j) Give a general description of how an unbound nuclear state would decay.

An unbound nuclear state should decay by the emission of a neutron, proton or other massive particle. These decays would almost always occur faster than beta or gamma decay.

2. (15 points) ^{10}B is used as a neutron absorbing material in a variety of applications due to its high cross section for the (n,α) reaction. This reaction actually produces the product ^{7}Li in the nuclear ground state and in its first excited state. (A) What does the simple nuclear shell model predict for the ground state nucleon configurations for this nucleus? (B) What is the predicted spin and parity of the ground state of ^{7}Li? (C) What does this model predict for the spin and parity of the first excited state of ^{7}Li? (D) How would the predictions of the simple shell model for the states in ^{7}Be compare to the predicted states in ^{7}Li?

(A)$^{7}Li_4 : \pi(1s^2_{1/2}1p^1_{3/2}), \nu(1s^2_{1/2}1p^2_{3/2})$
(B) odd nucleon (a proton) is in $p_{3/2}$ orbit .. j=3/2, parity = odd
(C) smallest energy to promote the odd-proton into p$_{1/2}$ orbit .. j=1/2, parity =odd
(D) $^{7}Li_4$ and $^{7}Be_3$ are "mirrors" – the energy level pattern should be the same.

3. (15 points) The vast majority of the material in our solar system (and the universe) is H and He that is thought to have been made in the big bang and remains since that time. An extremely simple picture of the big bang is a high energy explosion in which neutrons and protons have sufficient kinetic energy to undergo all sorts of (sequential) nuclear reactions before the whole system blows apart and the reactions stop. (A) Based on the nuclear shell model, explain why heavier elements (Z>2) could not be produced in the big bang. (B) What is the source of energy for our sun? (C) In contrast to the solar system, the most abundant isotope on earth is Oxygen. Can oxygen be produced in the sun? If no, explain why not; if yes, explain how.

(A) There is a large gap between the 1s-shell and the 1p-shell. There are no stable nuclei with A=5 so fusion reactions would have to "jump" over this gap. Not very likely because the nuclei are being built-up by capture of neutrons and/or protons while the system is exploding. It is thought to come apart before it can make heavier nuclei.
(B) The sun is powered by the conversion of four protons into one helium *atom* **plus two antineutrinos.**
(C) Oxygen cannot be made from helium in our sun due to: (1) gaps in stability at A=5 and A=8 that sequential reactions cannot jump, and (2) the temperature is not high enough to fuse three helium nuclei into carbon (^{8}Be is unbound). HOWEVER, for purists, some oxygen is "made" as a reaction intermediate in the catalytic CNO cycle that converts protons into helium using carbon nuclei as seed nuclei. The carbon nuclei are thought to have been produced elsewhere before the formation of the sun and the solar

system (the same process that made the oxygen on the earth).